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THE JOULE-THOMSON PROCESS IN CRYOGENIC REFRIGERATION SYSTEMS

J. W. Dean and D. B. Mann

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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A comprehensive analysis of the Joule-Thomson process as applied to cryogenic refrigeration systems is presented. The descriptions of the process already in the literature are usually for specific applications. In contrast, performance characteristics are presented here for helium, para-hydrogen, and nitrogen operations over a large range of process parameters.

1. INTRODUCTION

The objective of this report is to present a comprehensive analysis of the Joule-Thomson process as applied to cryogenic refrigeration systems. The descriptions of this process already in the literature [1], [2] are usually for specific applications. In contrast, performance characteristics are presented here for helium, para-hydrogen, and nitrogen operation over a large range of process parameters.

Energy and flow rate requirements are given for the Joule-Thomson refrigeration process operating in a system consisting of a heat exchanger, an expansion valve, and an evaporator. These components are often found in the lowest temperature portion of a refrigeration system with precooling provided by a liquid bath or an expansion engine. Therefore, the information contained in this report may be combined with performance characteristics of other components to find the overall energy requirements of a more complex refrigeration system. For example, liquid nitrogen use rate and cost information may be combined with data from this work to obtain the overall energy requirement for a nitrogen precooled hydrogen refrigerator. In addition, by using the charts which follow, a complete refrigeration system can be constructed by cascading the

Joule-Thomson process. It is expected that the graphs and tables of this report will be helpful to the engineer in optimizing his design or evaluating an existing design.

The data presented in this report represent the results of many process calculations that were made possible by the use of an electronic digital computer.

2. NOMENCLATURE

P	-	pressure - atm.
ρ	-	density - gm/liter
T	-	temperature - °K
H	-	enthalpy - joules/gm
$\dot{m}$	-	mass flow rate - gm/sec
Q	-	refrigeration - watts
v	-	specific volume - liters/gm
WC	-	rate of compressor work -watts
R	-	universal gas constant - 8.3147 J/°K-mole
SCFM	-	standard cubic feet per minute defined at 760 mm. Hg and 0°C
M	-	gram molecular weight
η_c	-	isothermal compression efficiency
μ	-	Joule-Thomson coefficient - °K/atm.
C_p	-	specific heat - J/gm-°K

Numerical subscripts refer to the state of the gas as shown on the temperature - entropy diagram of figure 5.

amb. - refers to the ambient temperature of the compressor, taken as 300°K.

3. PROPERTIES OF FLUIDS

The refrigerants used for this analysis are helium, para-hydrogen, and nitrogen. Isothermal refrigeration is made available by the evaporation of these fluids from the liquid state at their respective normal boiling point temperatures of 4.21°K, 20.268°K, and 77.36°K.

Thermodynamic properties of these fluids have been generated by the National Bureau of Standards-Cryogenics Division in order to obtain thermodynamically consistent data of high accuracy and are now available in the literature [3, 4, 5]. The thermodynamic data tabulated at 1°K increments for integer values of pressure were put on magnetic tape for high speed input to the computer. These data have been generated by the manipulation of the state equations given in the above referenced works. Functional relationships, derived for the generation of properties, were used in the refrigeration analysis, when data at non-integer values of pressure were required, and are:

$$P = f(\rho, T),\ \rho = f(P, T),\ H = f(\rho, T),\ (\partial P/\partial \rho)_T \text{ and } (\partial P/\partial T)_\rho .$$

Data for normal hydrogen [6] had been generated, but were not used in the refrigeration analysis. If a constant ortho-para fraction is established in the hydrogen refrigerant throughout the process, the analysis is such that the actual ortho-para fraction does not effect the results. Para-hydrogen was used because of the greater range of available data.

Since the Joule-Thomson refrigeration process exploits the cooling effect obtained by expansion of gases under certain conditions, it is necessary to establish the conditions where cooling occurs. The inversion curve for each of the gases is shown in figures 1, 2, and 3. These curves were obtained by calculation from the state equations [3, 4, 5] in order to be consistent with the refrigeration process data presented in this report.

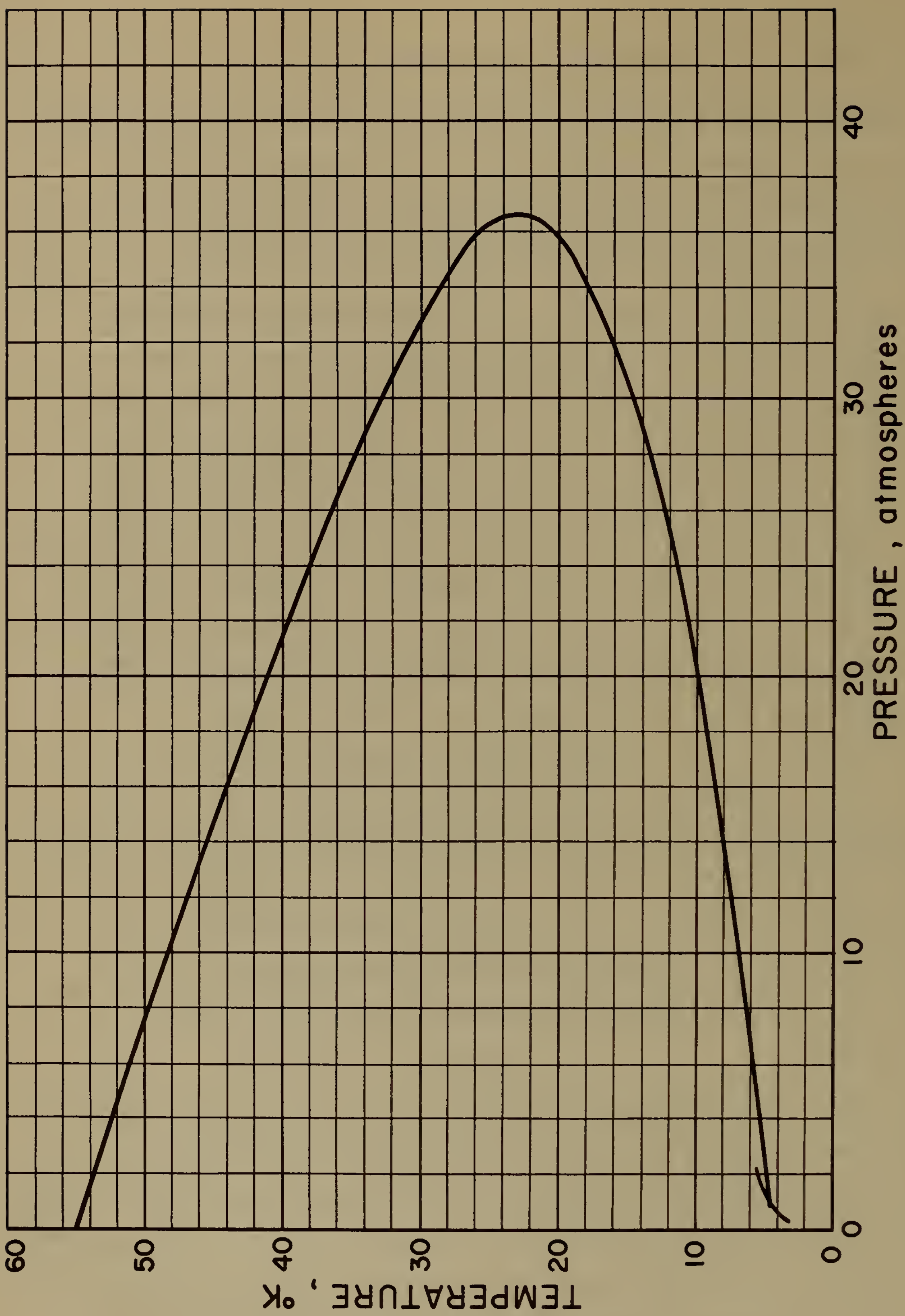

Figure 1. Helium Inversion Curve

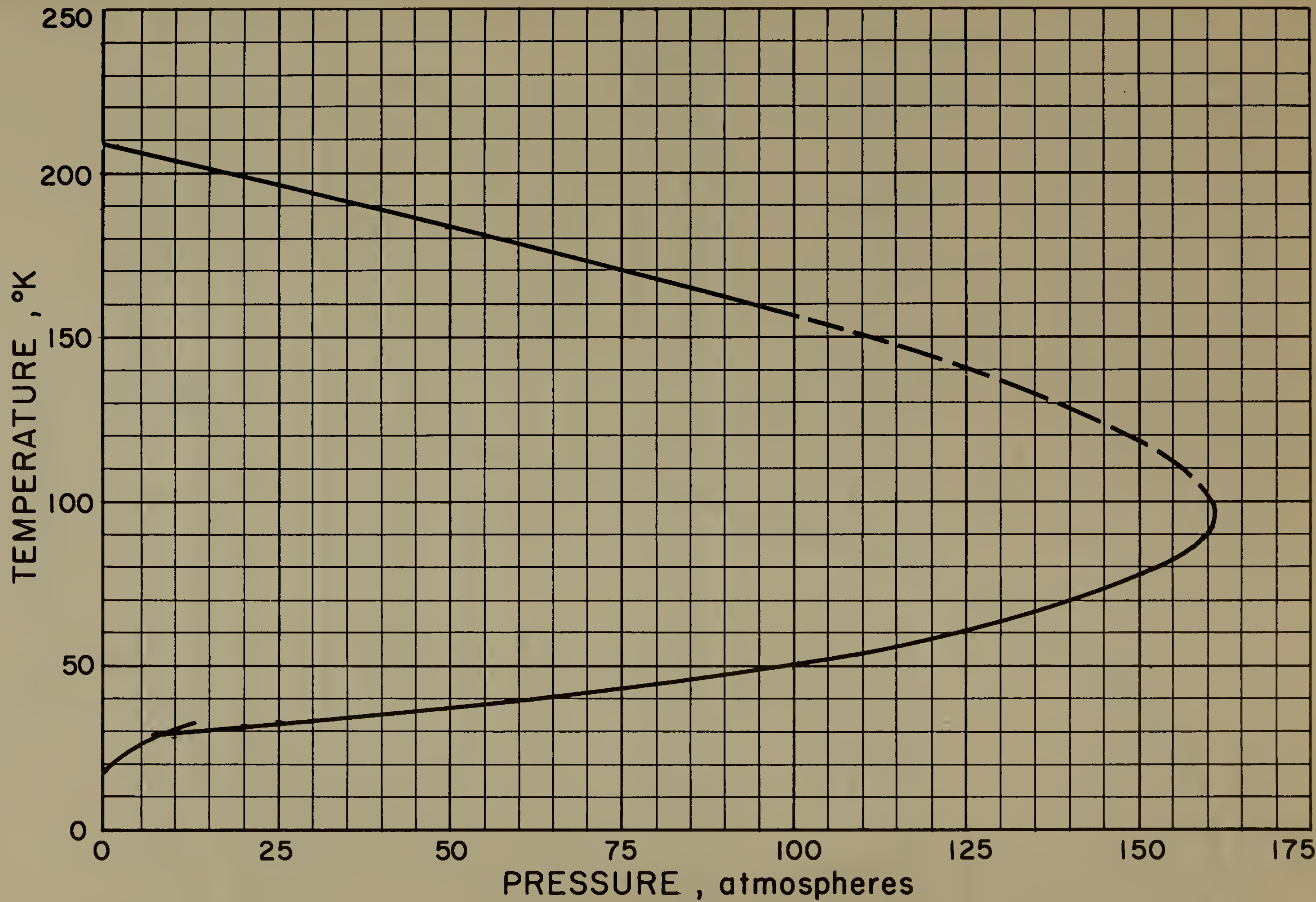

Figure 2. Para-Hydrogen Inversion Curve

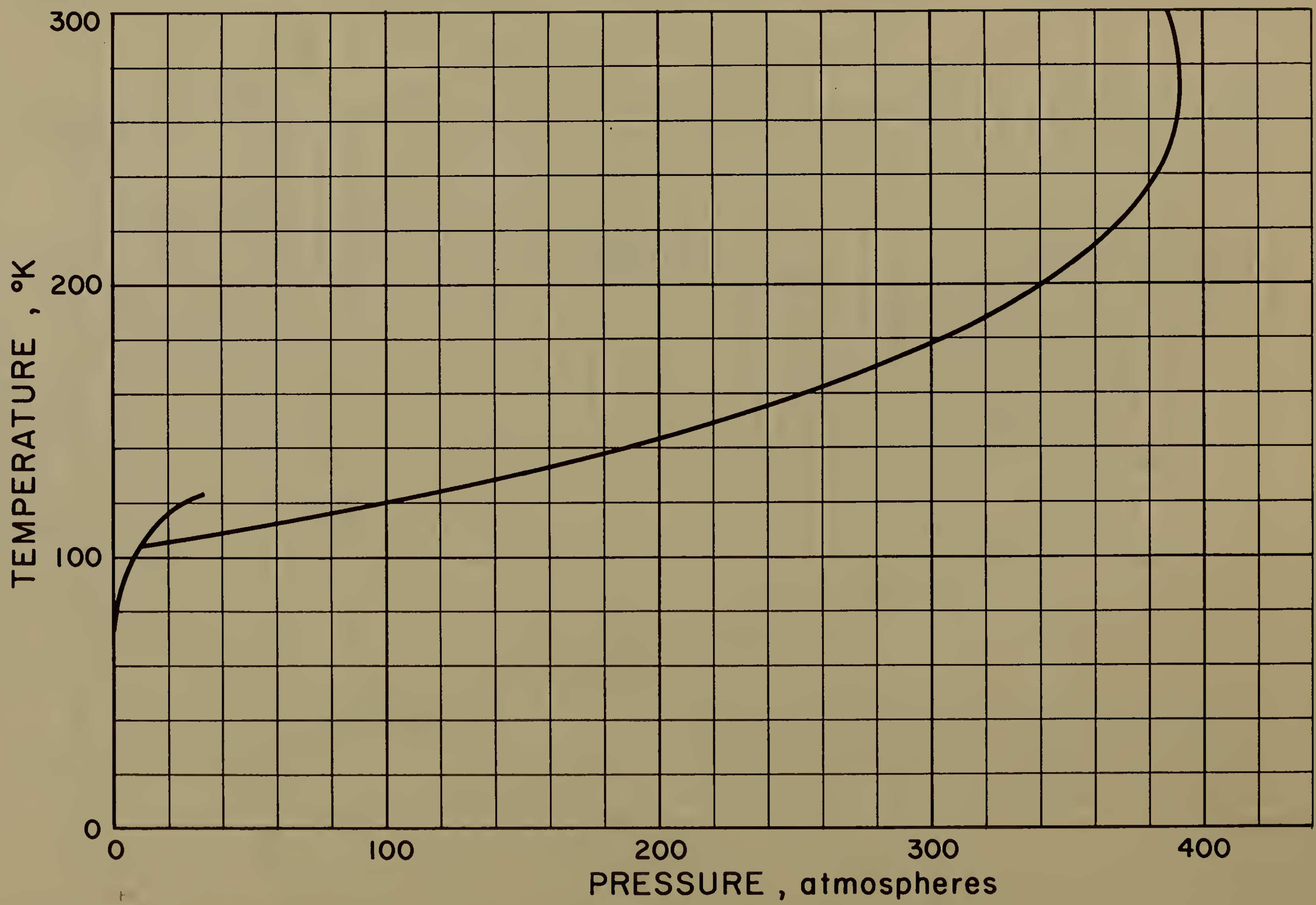

Figure 3. Nitrogen Inversion Curve

Calculations of the pressure and temperature locus for the inversion curves were performed by satisfying the condition that the slopes of the isenthalps be zero. These slopes are given by the Joule-Thomson coefficient, $\mu = (\partial T/\partial P)_H$, and may be defined in terms of the physical properties by the expression [7]

$$\mu = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T}\right)_P - v \right].$$

In order to satisfy the condition that the Joule-Thomson coefficient be zero, it is necessary and sufficient that

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{v}{T} . \tag{1}$$

Equation (1) was modified to be consistent with the state equations and their derivatives resulted in the following form:

$$\frac{\left(\frac{\partial P}{\partial T}\right)_\rho}{\left(\frac{\partial P}{\partial \rho}\right)_T} - \frac{\rho}{T} = 0 . \tag{2}$$

The locus of points satisfying (2) was found for temperatures ranging from near the critical to the maximum inversion temperature. Inversion pressures were found to a precision of 0. 1 atmospheres. Values of pressure and temperature for the inversion curves obtained from the state equations were found to be in good agreement with inversion curve data of the literature [7, 8, 9, 10]. Dashed portions of the inversion curves indicate regions of extrapolation.

4. JOULE-THOMSON EXPANSION PROCESS

Regions of heating and cooling are clearly shown when the inversion curve is plotted on temperature pressure coordinates. A Joule-Thomson expansion process which has initial and end states within the inversion curve envelope (figure 4) will lower the temperature of the fluid while such a process occurring outside the envelope will raise the temperature of the fluid. Expansion from a temperature above the maximum inversion temperature will always result in heating. If the fluid is expanded from state A to state B, the temperature of the fluid will rise. Further expansion to state C results in cooling to the original temperature and an end state of D will lower the temperature below the initial temperature. Since the same net temperature drop is obtained by expanding the gas from either states C or A to state D, there is no reason for expending the energy required to compress the fluid to any pressure higher than the pressure at C.

Lowering the initial temperature results in operation in the portion of the temperature-pressure plane where the slopes of the isenthalps are greater and eventually where the isenthalps intersect the vapor pressure curve. This intersection is the extension of the isenthalps into the two phase liquid-vapor region and means that a fraction of the gas expanded into this region is liquefied.

State D' is the intersection of the isenthalp with the two phase liquid vapor region and E' is the intersection of the isenthalp with the final pressure isobar. The choice of the final pressure corresponding to E' establishes the temperature of the liquid following the expansion process. The pressures at D' and E' should be as low as possible if maximum temperature drop is desired.

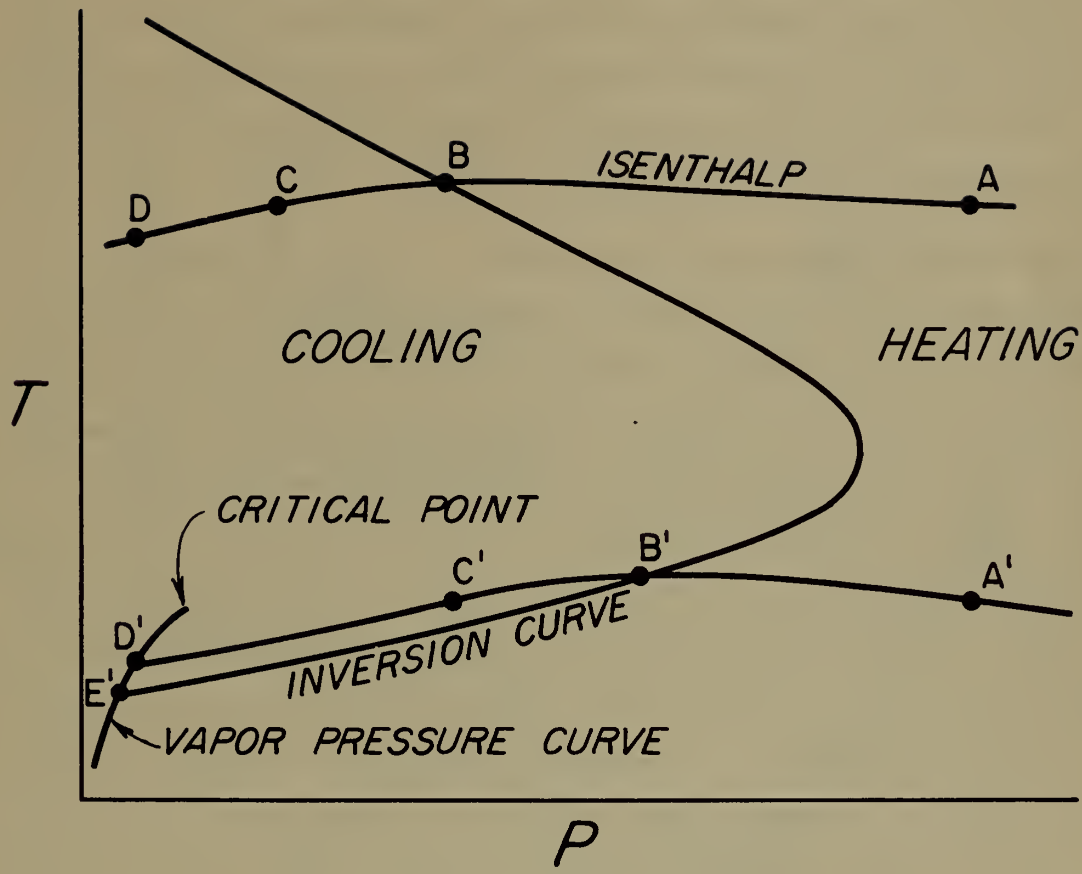

Figure 4. Joule-Thomson Expansion Process

5. PROCESS DISCUSSION

The Joule-Thomson refrigeration system is defined as a counter-flow regenerative heat exchanger connected in series with an expansion valve and an evaporator as shown schematically in figure 5. The compressor for the system operates at ambient temperature while the heat exchanger of figure 5 operates generally in the cryogenic temperature range. Intermediate heat exchangers and expansion engines are not considered as part of the thermodynamic system.

The system may be considered to be at a uniform temperature before starting. This may be the case when a liquid precoolant bath is in contact with, but outside of the system. Under this condition, gas expansion at the valve acts as discussed previously under the expansion process. Thus, appropriate precooling temperature and operating pressure choices must be made for each refrigerant considered. Starting the refrigeration process consists of providing a quantity of compressed gas to point 1. The refrigeration made available by the expansion process is used to cool the mass of metal of the evaporator and the return stream side of the heat exchanger. The external heat load is withheld in order to expedite cooling. Cooling the heat exchanger return stream side, between points 4 and 5, results in cooling the inlet gas stream between points 1 and 2. For the ideal heat exchanger considered here, points 1 and 5 remain at the initial temperature while the temperature of points 2, 3 and 4 are depressed. As the temperature of point 2 is depressed, the temperature of points 3 and 4 also become colder due to the Joule-Thomson effect. Thus the refrigeration process is self-perpetuating until an equilibrium condition is reached.

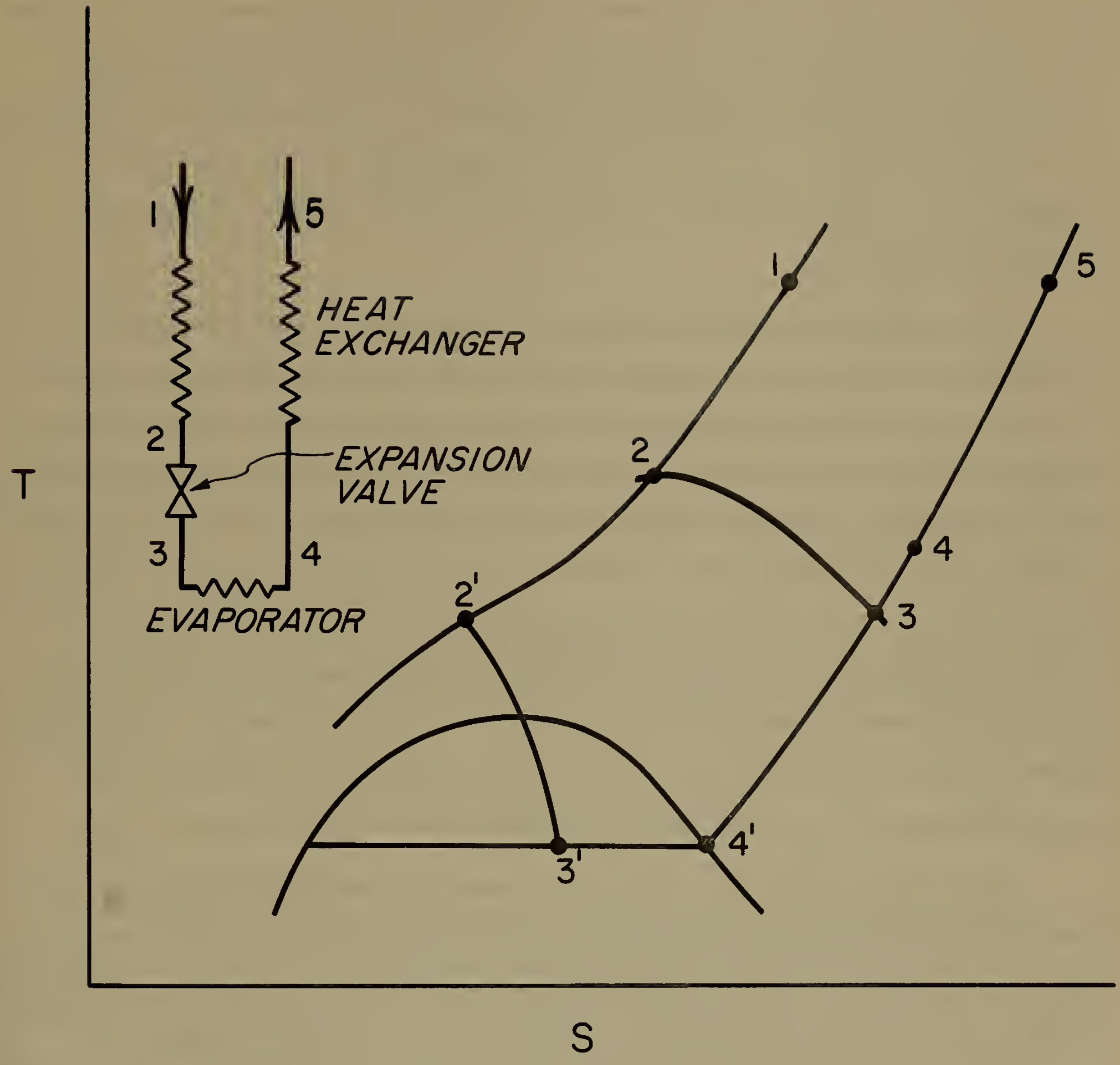

Figure 5. Schematic of Refrigeration System

A possible equilibrium condition is shown by points 1, 2, 3, 4, and 5 on the temperature-entropy surface of figure 5. An external heat load has been applied to the evaporator between points 3 and 4 and is just equal to the refrigeration available. No refrigeration is available to further cool the heat exchanger. No condensation of liquid occurs; under this condition the term "evaporator" is a misnomer as refrigeration is obtained non-isothermally from the sensible heat of the gas. This condition is difficult to maintain because of the low thermal capacitance of the gas in the evaporator. Small variations in the heat load will rapidly change the system temperature.

The stable equilibrium condition at which the Joule-Thomson refrigeration process is normally operated is shown by points 1, 2', 3', 4' and 5 on the temperature-entropy surface of figure 5. Temperature depression at the expansion valve continues until liquid is condensed into the evaporator. Refrigeration is obtained isothermally by the evaporation of the liquid. A prerequisite for this type of operation is that the return stream pressure be less than critical pressure since the temperature at which the evaporator will operate is fixed by this pressure as defined by the vapor pressure curve of the refrigerant. Operation with a one atmosphere return stream pressure will result in evaporator temperatures as given under the normal boiling temperatures in the section on data.

This type of operation tends to be stable in temperature due to the presence of liquid and the magnitude of the heat of vaporization. Mass flow through the evaporator must be matched to the heat load for continuous stable operation as an excessive heat load will result in reducing the quantity of liquid rather than changing the system temperature. If corrective action is not taken, such as increasing the compressor flow rate, the evaporator will eventually go dry forcing the evaporator temperature to increase into the non-isothermal operating region. In contrast, insufficient heat load will result in flooding of any type of evaporator. Thus, the refrigeration process must be controlled in either mode, but the case of isothermal operation is easier to control.

6. METHOD OF PERFORMANCE RATING

If the coefficient of performance method of rating refrigerators is applied to cryogenic systems, the results are performance figures less than unity. Here the performance has been rated by the reciprocal of the coefficient of performance and is defined as the compression power requirement divided by the refrigeration absorbed. Actual calculations of refrigeration and power were performed in the metric system using joules and seconds as energy and time units; however, the performance rating is a dimensionless number always greater than unity and may be considered as the ratio of the watts of electrical power provided to the compressor motor at ambient temperature to watts of refrigeration available at cryogenic temperature.

The power required for the compressor was assumed to be equal to the power required to compress an ideal gas isothermally between the same states:

$$WC = -\dot{m} \frac{R\,T_{amb}}{M} \ln (P_1/P_5) \; . \qquad (3)$$

To obtain realistic refrigeration power requirements, the power term as calculated by (3) must be divided by the isothermal compression efficiency--a figure of 60 to 70 percent is common for reciprocating compressors using piston rings. The useful refrigeration, Q, absorbed at cryogenic temperature was calculated from first law energy balance considerations. Performance is rated by the ratio WC/Q, a dimensionless figure of merit. In order to put the calculations on a common basis, division by the isothermal efficiency has not been performed and is left to the reader who must consider the merits of his compressor. When a refrigeration system is considered that utilizes other energy sources, such as an auxiliary refrigerator, that power term must be added to the numerator to obtain an overall performance figure.

Refrigeration performance may also be stated in terms of theoretical work as found by the Carnot principle. The Carnot work may be expressed as a dimensionless number by the ratio of the work input to the quantity of heat received at the lowest temperature. The expression for the Carnot work per unit of refrigeration available is

$$\text{Carnot Work} = \frac{T_{amb} - T'_3}{T'_3}$$

and gives the limit to the performance of a refrigerator between the stated temperatures.

A method of rating cryogenic refrigerators now in some use is to compare the ratio of the Carnot work to the measured or calculated work considering the irreversabilities. Thus, a process efficiency is stated in terms of percent Carnot as

$$\text{Percent Carnot} = \frac{\dfrac{T_{amb} - T'_3}{T'_3}}{\dfrac{WC}{\eta_c Q}} \times 100.0 \; . \qquad (4)$$

A result of 25 percent Carnot, meaning that the calculated or measured work is four times theoretical, is a possible result. If care is not taken, this method leads to some difficulty when non-isothermal refrigeration occurs. Under this condition, the Carnot work must be defined by the more general expression [11]:

$$\text{Carnot Work} = \frac{T_{amb}}{(T_4 - T_3)/\ln(T_4/T_3)} - 1 . \tag{5}$$

The percent Carnot expression for non-isothermal refrigeration then becomes:

$$\text{Percent Carnot} = \frac{\dfrac{T_{amb}}{(T_4 - T_3)/\ln(T_4/T_3)} - 1}{\dfrac{WC}{\eta_c Q}} \times 100.0 . \tag{6}$$

Difficulty arises with the percent Carnot method of performance rating when the distinction is not made between equations (4) and (6). When superheating occurs, a portion of the evaporator is isothermal while the remainder is not. To use the percent Carnot performance rating method under this condition requires the combination of equations (4) and (6). To avoid the ambiguity of the percent Carnot approach, this work uses the performance rating of WC/Q.

7. CALCULATIONS

Application of the first law of thermodynamics to the refrigeration system of figure 5 where frictional losses due to velocity effects are negligible yields the following energy balance:

$$Q = \dot{m}\ (H_5 - H_1) . \tag{7}$$

The externally useful refrigeration is equal to the difference in the total enthalpy of the refrigerant entering and leaving the system at the top of the heat exchanger-points 1 and 5. The temperature at which the useful refrigeration is available is not explicitly defined by (7) and may range from near the boiling temperature of the refrigerant to near the precooling temperature. Equation (7) is valid for the isothermal and non-isothermal mode of refrigerator operation and will yield identical results for the same state of the refrigerant at points 1 and 5. Under these conditions the work of compression is also the same. The performance rating, WC/Q, is therefore independent of the mode of refrigerator operation and depends only on the state of the refrigerant at points 1 and 5.

The above statement seems to be incongruous in view of the Carnot principle; the compressor is operated at ambient temperature while the evaporator temperature level is varied, yet at the same time the WC/Q remains unchanged. Compressor power requirements are based on the conditions of (7), and these conditions are independent of the evaporator operating temperature. Therefore, the figure of merit, WC/Q,is insensitive to changes in the evaporator temperature once states 5 and 1 have been found. Operation in the non-isothermal mode of refrigeration does not allow the refrigeration system to achieve its low temperature capability; therefore, non-isothermal operation of the Joule-Thomson refrigeration system is normally not desired. The WC/Q figure of this work, although valid for the non-isothermal operating mode, should be considered for isothermal refrigeration.

Calculations were made on the digital computer satisfying equations (3) and (7) for a wide range of parameters consisting of the heat exchanger operating pressures, precooling temperature, and temperature difference at the top of the heat exchanger. The majority of the calculations were made with a return stream pressure of one atmosphere allowing isothermal refrigeration at the normal boiling point temperature. Inlet stream pressure and precooling temperature were varied from near the critical to near the maximum inversion pressure and temperature. The temperature difference at the top of the heat exchanger was varied by reducing the temperature at point 5. The figure of merit and the flow rate in gram moles per second required to produce one watt of useful refrigeration was calculated for each set of parameters.

Temperature differences between heat exchanger refrigerant streams introduce irreversabilities, but they are necessary to effect heat transfer. Consider the equations which describe the heat transfer between the two gas streams of the heat exchanger:

$$\text{heat exchanger load} = \dot{m} \int_{T_1}^{a} Cp_{1-2}\, dT = \dot{m} \int_{T_5}^{b} Cp_{4-5}\, dT, \quad (8)$$

where a and b are intermediate temperatures of the high and low pressure gas streams (see figure 6). The temperature and pressure dependency of the specific heat allows the possibility that $b \geq a$ for small values of $T_1 - T_5$. Since b represents the temperature of the available refrigeration of the low pressure stream, the condition $b \geq a$ is not consistant with the second law concept of heat energy flow and is therefore indicative of an invalid choice of operating parameters. Either $T_1 - T_5$ must be increased or other operating pressures and precooling temperatures must be chosen. Increasing $T_1 - T_5$ without changing the precoolant temperature will always increase WC/Q because of the temperature dependency of the term $(H_1 - H_5)$ in (7). The designer should investigate the temperature distribution within the heat exchanger for both gas streams from (8). Even though a minimal temperature difference occurs within or at the cold end $(T_2 - T_4)$ of the exchanger, the temperature difference $T_1 - T_5$ may be used as a parameter to indicate heat exchanger energy losses. Once this temperature difference is fixed, then the internal temperature differences become a function of the properties of the gas.

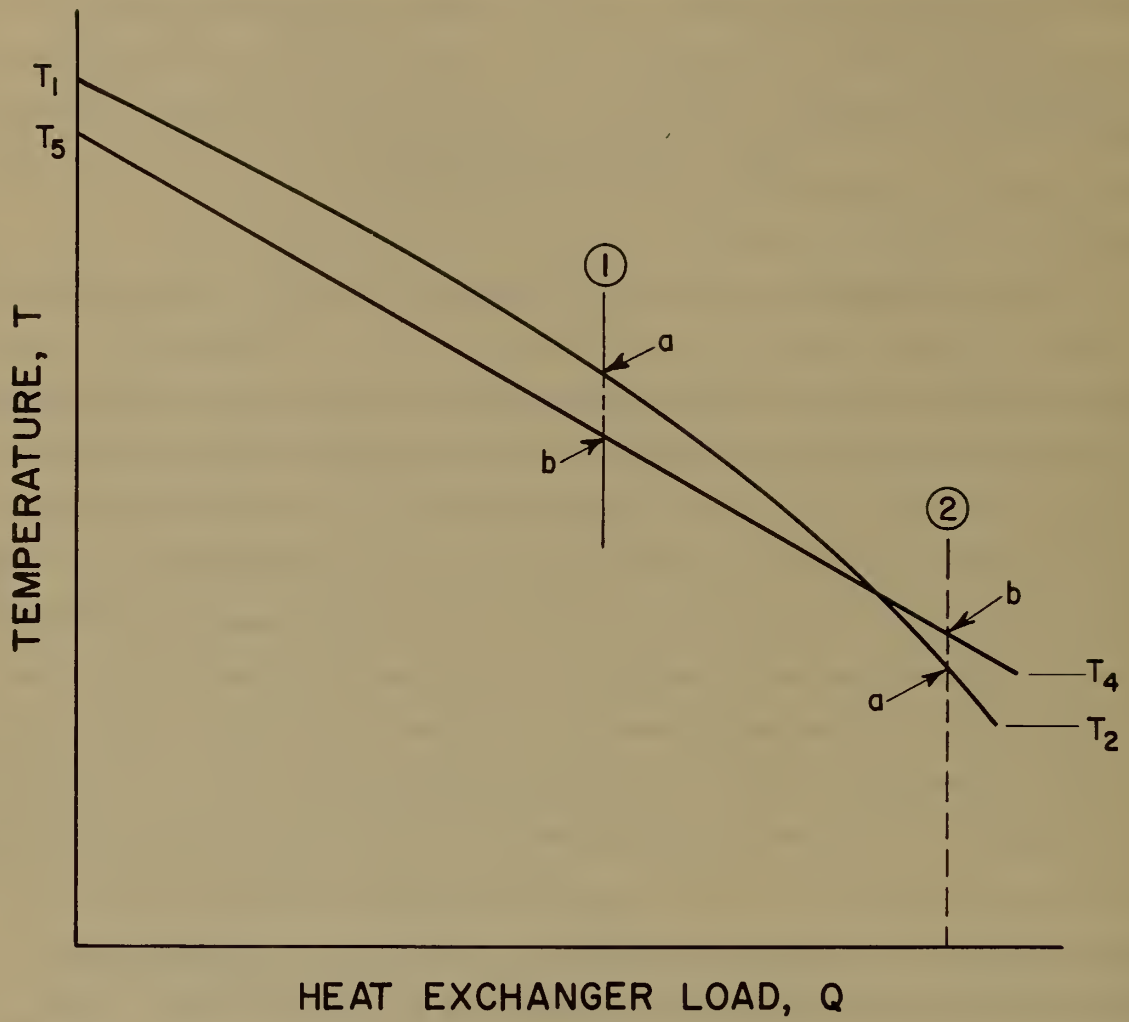

Figure 6. Heat Exchanger Cooling Curve - General Case

It is desirable to determine the combination of parameters that give both the maximum obtainable refrigeration for a given compressor size and the most economical operation. However, no practical single set of parameters satisfies both requirements.

An examination of figure 4 has shown that the maximum refrigeration is obtained by an expansion process when the inital state of the gas before expansion is on the inversion curve. The effect of inserting an evaporator and a heat exchanger in the system is not so easily seen. Consider equation (7). The pressure, temperature, and mass flow rate at point 5 and the temperature and flow rate at point 1 may be considered constant for an equilibrium condition. Thus, the only variable is $H_1 = f(P)$, and the condition for maximum refrigeration becomes $\left(\frac{\partial H_1}{\partial P}\right)_T = 0$. Using an elementary relationship for partial derivities, this condition may be restated as

$$\left(\frac{\partial H_1}{\partial T}\right)_P \bullet \left(\frac{\partial T}{\partial P}\right)_H = 0$$

or

$$Cp \bullet \mu = 0 . \qquad (9)$$

The Joule-Thomson coefficient must be zero in order to satisfy (9) as Cp is never zero through the operating range of the Joule-Thomson process. Thus, to obtain maximum refrigeration for a fixed mass flow rate and precooling temperature, the inlet stream pressure must lie on the inversion curve.

Note that the above statement does not define the temperature of the expansion valve. In the case where T_1 and P_1 are chosen to lie on the lower portion of the inversion curve, the expansion valve temperature will be in the region of heating of figure 4 because of the temperature depressing effect of the heat exchanger. Of course, no heating will occur during the expansion as the Joule-Thomson process is undefined except for the end states which lie inside the cooling region of figure 4.

The minimal value of the figure of merit, WC/Q, for a fixed precooling temperature, temperature difference at the top of the heat exchanger, and return stream pressure indicates the most economic inlet stream pressure for refrigeration system operation. This inlet stream pressure may be found by plotting the results of calculations with equations (3) and (7). The resulting curves do not give clear minimum points. Therefore, an explicit expression was developed to determine these points.

The minimum figure of merit for the conditions described above is determined by

$$\left[\frac{\partial \left(\frac{WC}{Q} \right)}{\partial P_1} \right]_{T_1} = 0 , \tag{10}$$

substituting equations (3) and (7) into (10) and differentiating, yields

$$\left[\frac{\partial \left(\frac{WC}{Q} \right)}{\partial P_1} \right]_{T_1} = \frac{H_5 - H_1}{P_1} + \ell n \frac{P_1}{P_5} \left(\frac{\partial H_1}{\partial P_1} \right)_{T_1} = 0 . \tag{11}$$

The computer was programmed to solve equation (11) in an iterative manner for sets of precooling temperatures, return stream pressures, and heat exchanger temperature differences. The partial derivative was determined for trial P_1 and a new P_1 chosen to reduce the derivative toward zero. The optimum operating pressure was determined to a precision of 0. 1 atmospheres.

8. DISCUSSION

Figure 7 is an unscaled representation of the performance surface for a Joule-Thomson refrigeration system giving the figure of merit and flow rate as a function of inlet pressure for values of $T_1 - T_5$ with P_5 = one atmosphere. Continuous curves of WC/Q are shown for each precool temperature (T_1) from near the normal boiling temperature to near the maximum inversion temperature. The locus of the minimum WC/Q shown as calculated by (11) is marked "optimum," indicating the best operating pressures from the standpoint of power requirement. The inversion curve has been superimposed on the performance curve to indicate the states where maximum refrigeration is achieved as a function of mass flow.

Flow rate information , $\frac{\dot{m}}{QM}$, included with the performance curves by dashed lines, gives the flow in gram moles per second required to produce one watt of refrigeration. Since this is a flow rate divided by an energy rate, the time units cancel causing the flow information to have units of gm-mole/J. Multiplying these values by 47. 5 converts to cubic feet per minute (standard of 1 atm and 0°C) per watt of refrigeration--convenient units when dealing with American compressor manufacturers.

Results of (8) are given by a line marked "second law violation," indicating a limit to the range of possible operating parameters. Calculations of heat exchanger operation on or below this line result in $T_4 > T_2$ for isothermal refrigeration. Attempts to operate in this region result in $T_1 - T_5$ becoming larger than the designer anticipates with a corresponding reduction of performance--possibly to a point where no useful refrigeration is obtainable. One exception to this statement may be made if the refrigeration system is modified by inserting another heat exchanger and expansion valve in series as discussed in the literature [12, 13].

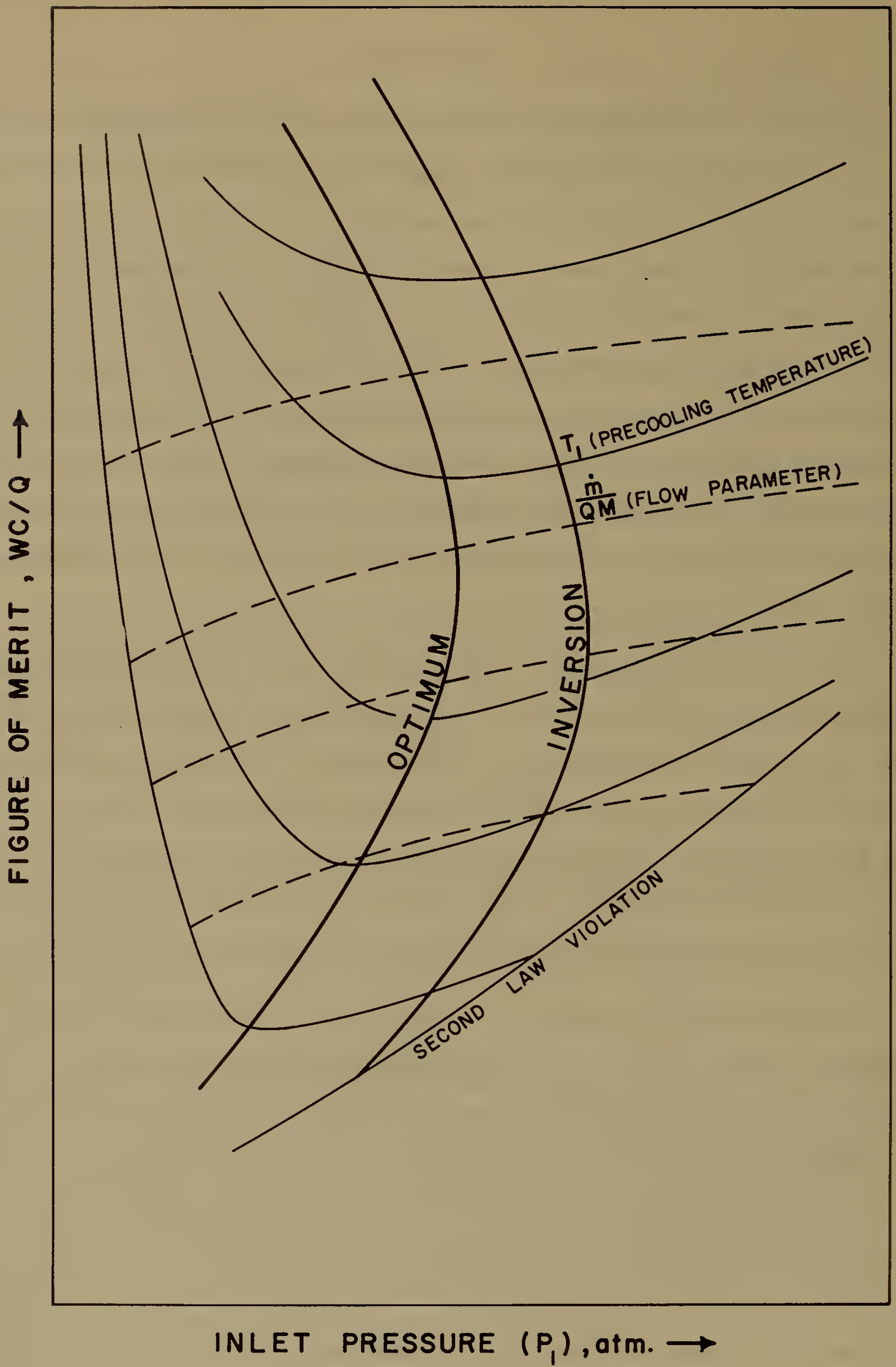

Figure 7. Generalized Performance Surface for a Joule-Thomson Refrigeration System

If an expansion valve was located in the high pressure stream at the location marked 1 in figure 6 and expansion allowed to take place to a pressure near, but slightly higher than the return stream pressure, then the temperature inversion at point 2 - or second law violation - might be avoided. The shape of the intermediate pressure cooling curve would become nearly that of the low pressure cooling curve. With care, the proper pressure might be found such that $a > b$.

Pressure drop within the heat exchanger cannot reduce the available refrigeration as it is a function of the gas state at points 1 and 5. However, heat exchanger pressure drop will effect the temperature at which the refrigeration takes place, since the pressure at 4 controls the isothermal refrigeration temperature as defined by the vapor pressure curve. Pressure changes at 5 within a region where the gas is nearly ideal -- $H_5 \neq f(P_5)$--will have little effect on the refrigeration but will effect the power requirement. WC/Q will decrease as the pressure at 5 is increased while the refrigeration temperature will increase as predicted by the relationship for Carnot work. An approximate correction to WC/Q values for a change in P_5 may be made by multiplying WC/Q by $(\ln P'_5/P)/(\ln P_5/P_1)$ where P'_5 is a value other than one atmosphere.

The general problem of pressure drop must be considered in a practical system irrespective of the foregoing discussion. Performance of the heat exchanger is based on the assumed temperature difference between points 1 and 5. Unless pressure drop is held to a minimum in both the high and low pressure streams, the temperature profile within the heat exchanger will be changed in such a way as to make the initially assumed temperatures unattainable.

Figures 8 through 20 show the performance surface for helium, para-hydrogen, and nitrogen refrigerants. Enough plots with different $T_1 - T_5$ values are given so that five percent accuracy may be achieved for WC/Q and flow values when linearly interpolating between plots in the lower half of these graphs. Interpolating at high precooling temperatures (T_1) should be conducted using a three point routine or crossplotting to enhance accuracy.

Optimum and inversion curves closely parallel each other and would appear as double valued functions if the performance data extended through a greater temperature range. Each gas having different characteristics allows different portions of the optimum and inversion curves to appear on the performance plot. The effect of increasing the value of $T_1 - T_5$ is to increase WC/Q and to shift the optimum curve to the right or to higher pressures. The helium gas case with $T_1 - T_5 = 1°K$ shows an effect where the optimum and inversion curves lie on the performance surface at very low WC/Q values and at low pressures. Performance curves for $T_1 = 6$ and $8°K$ show a positive slop indicative that these curves lie above the optimum pressure. This region of the graph represents a folded surface causing loss of clarity in data presentation with the complication of the possible crossing of the inversion and optimum curves.

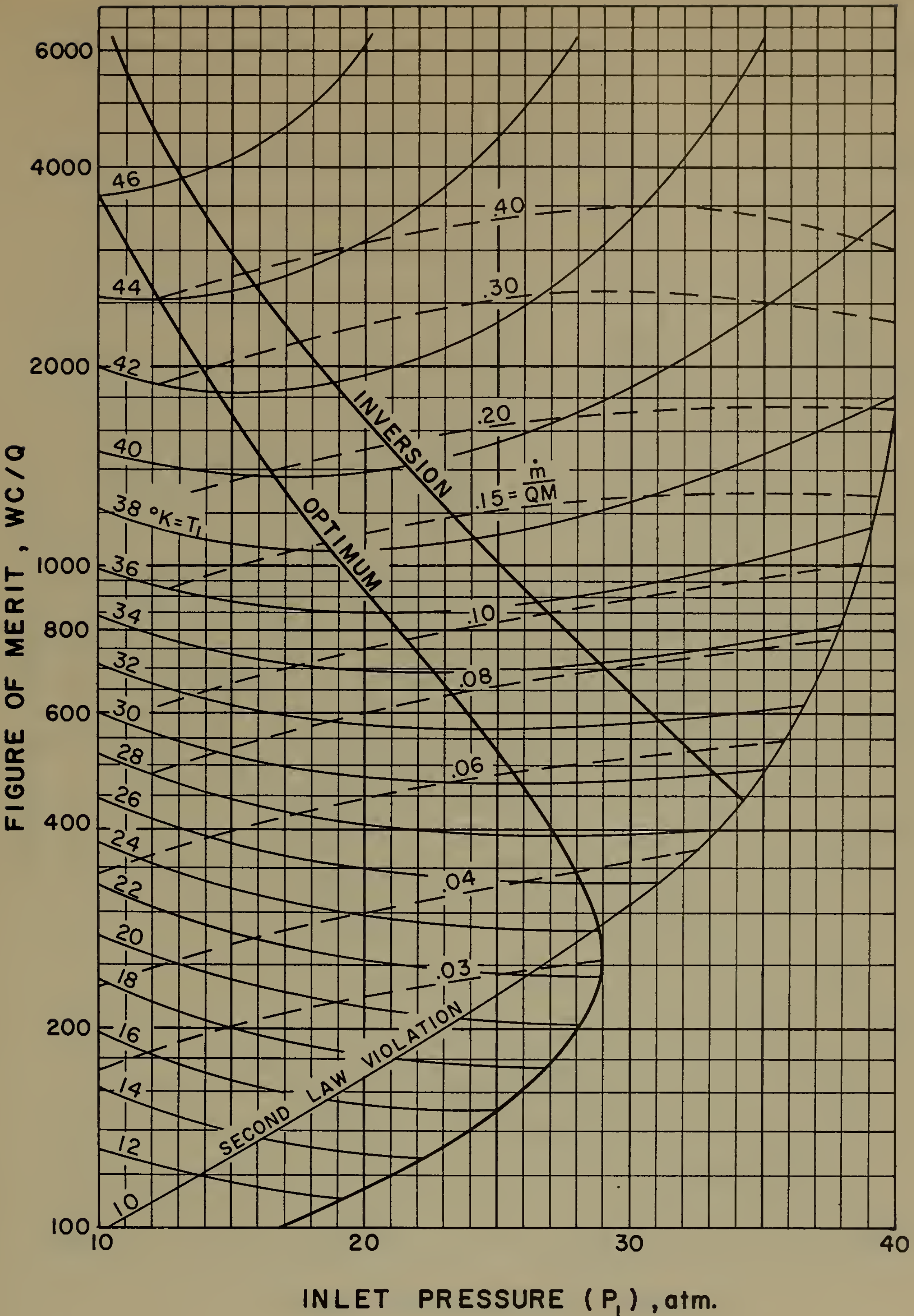

Figure 8. Refrigerator Performance - Helium Gas - $(T_1 - T_5) = 0.0°K$

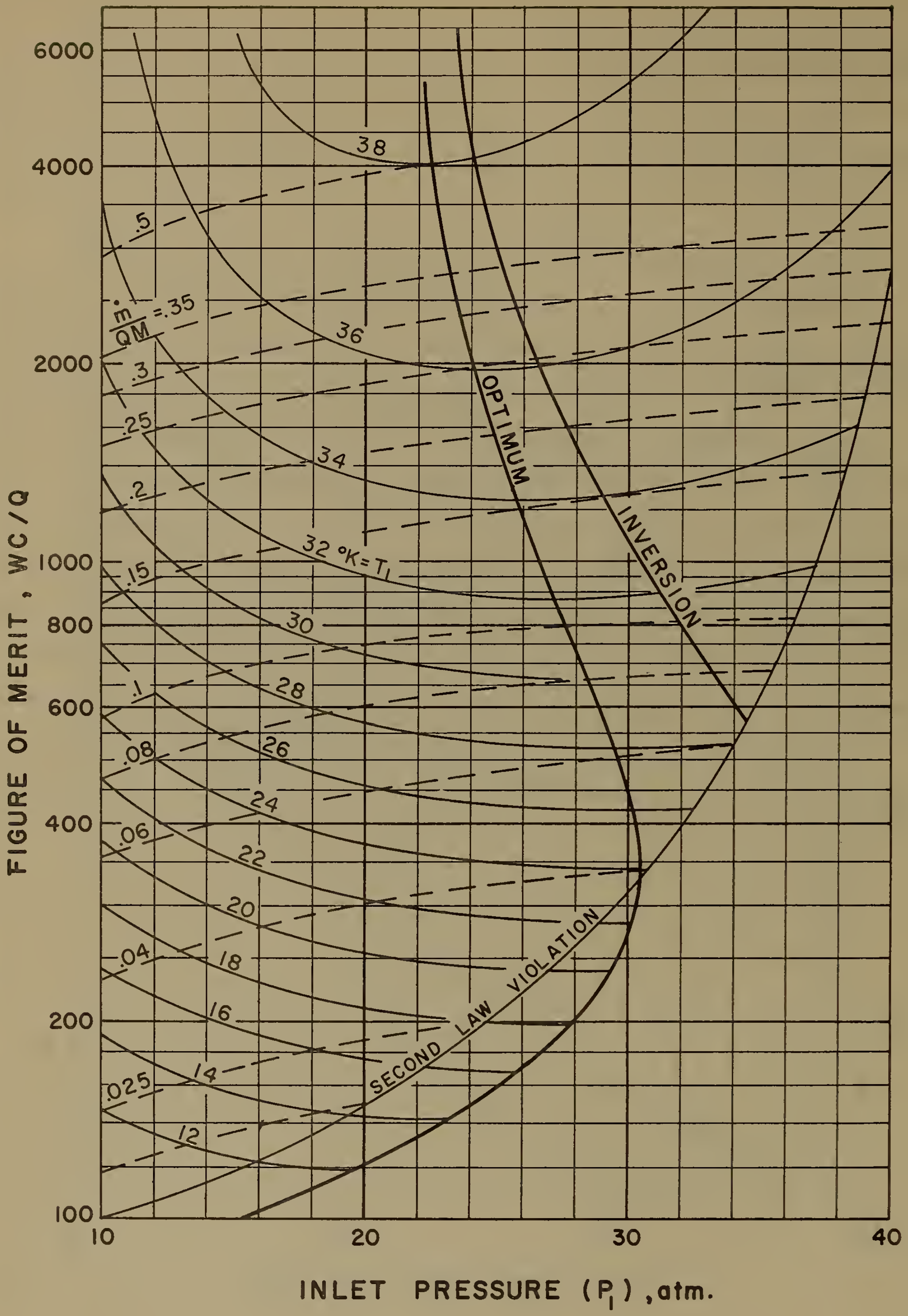

Figure 9. Refrigerator Performance - Helium Gas - $(T_1 - T_5) = 0.25°K$

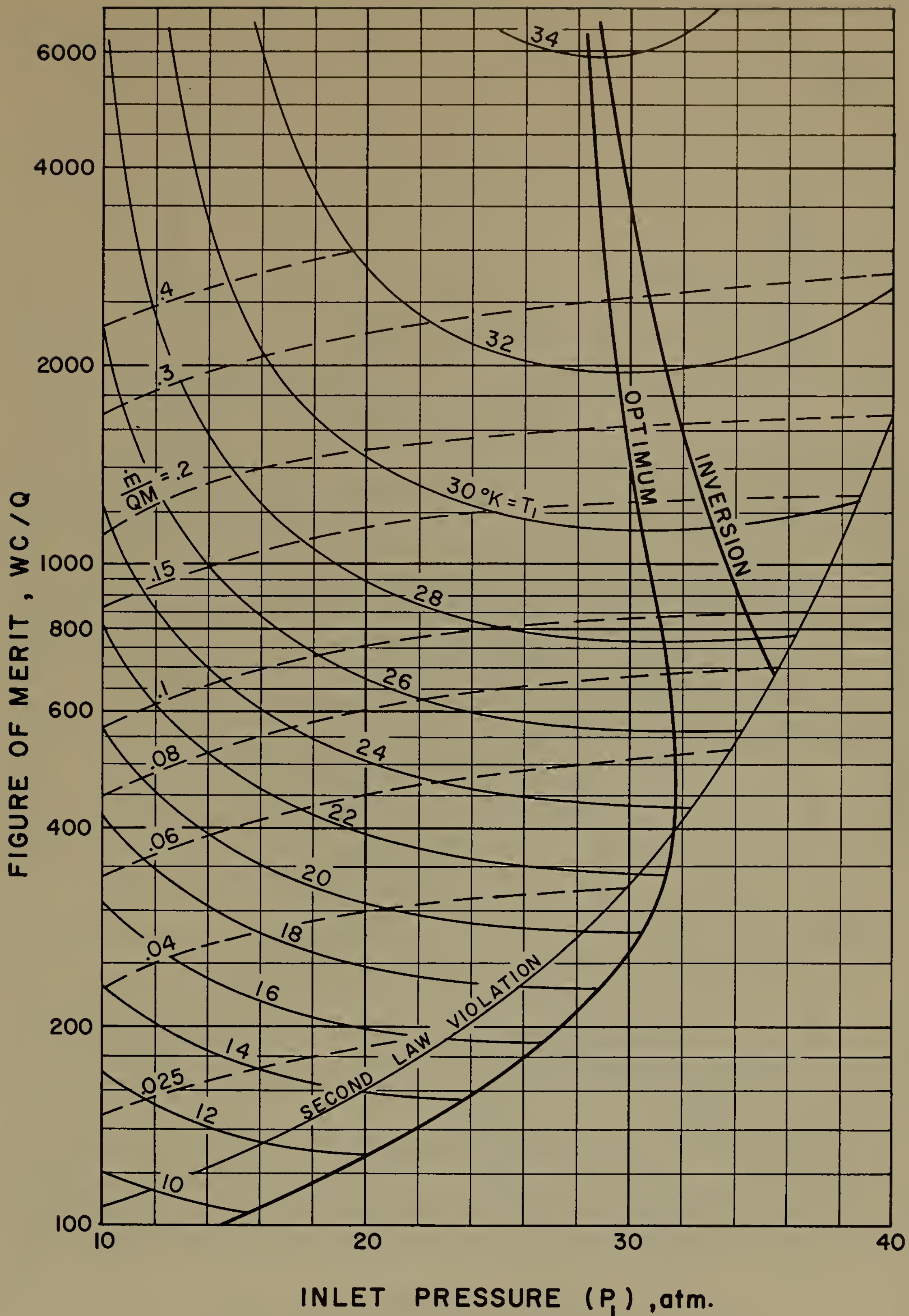

Figure 1.0. Refrigerator Performance - Helium Gas - $(T_1 - T_5) = 0.50°K$

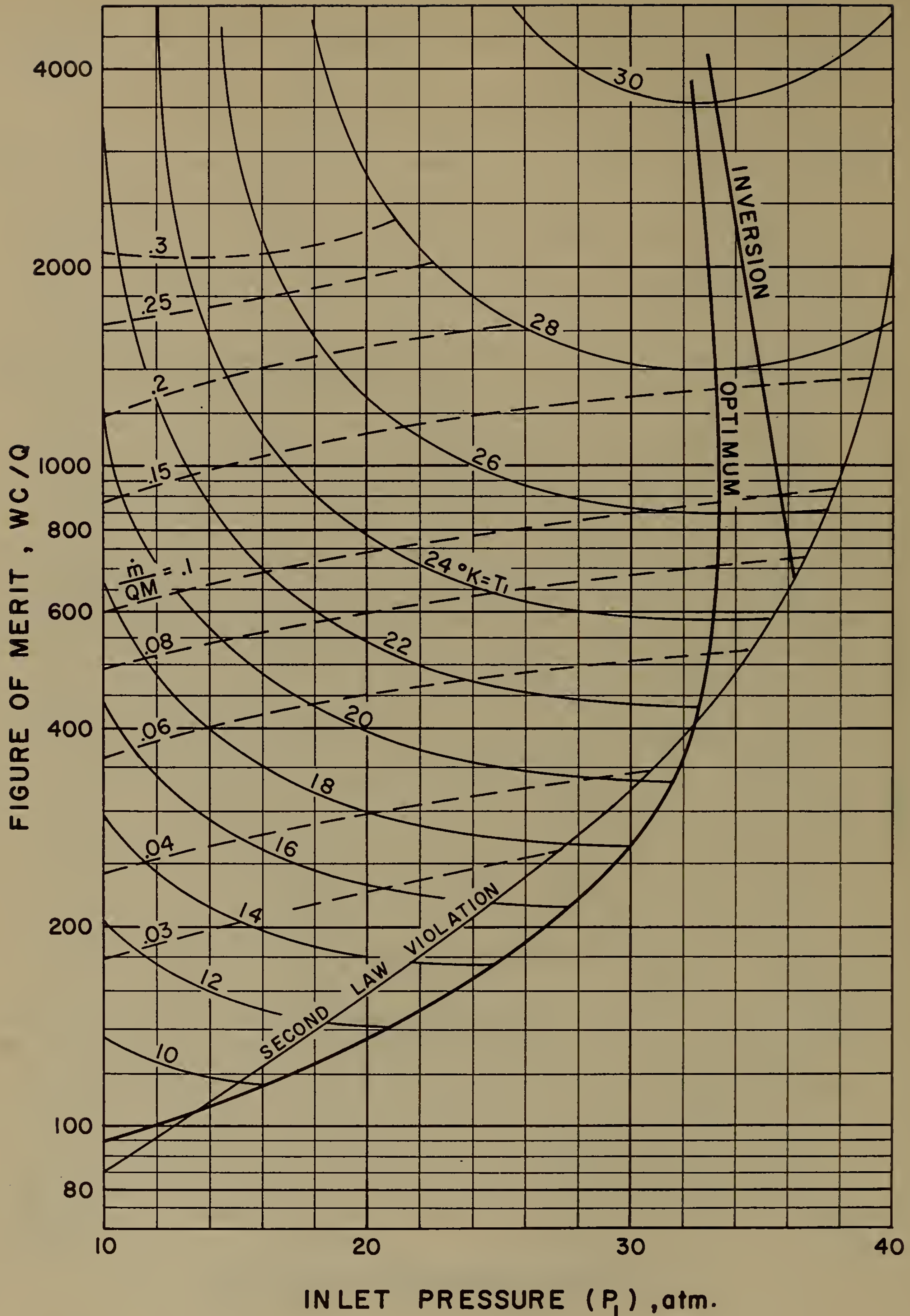

Figure 11. Refrigerator Performance - Helium Gas - $(T_1 - T_5) = 0.75°K$

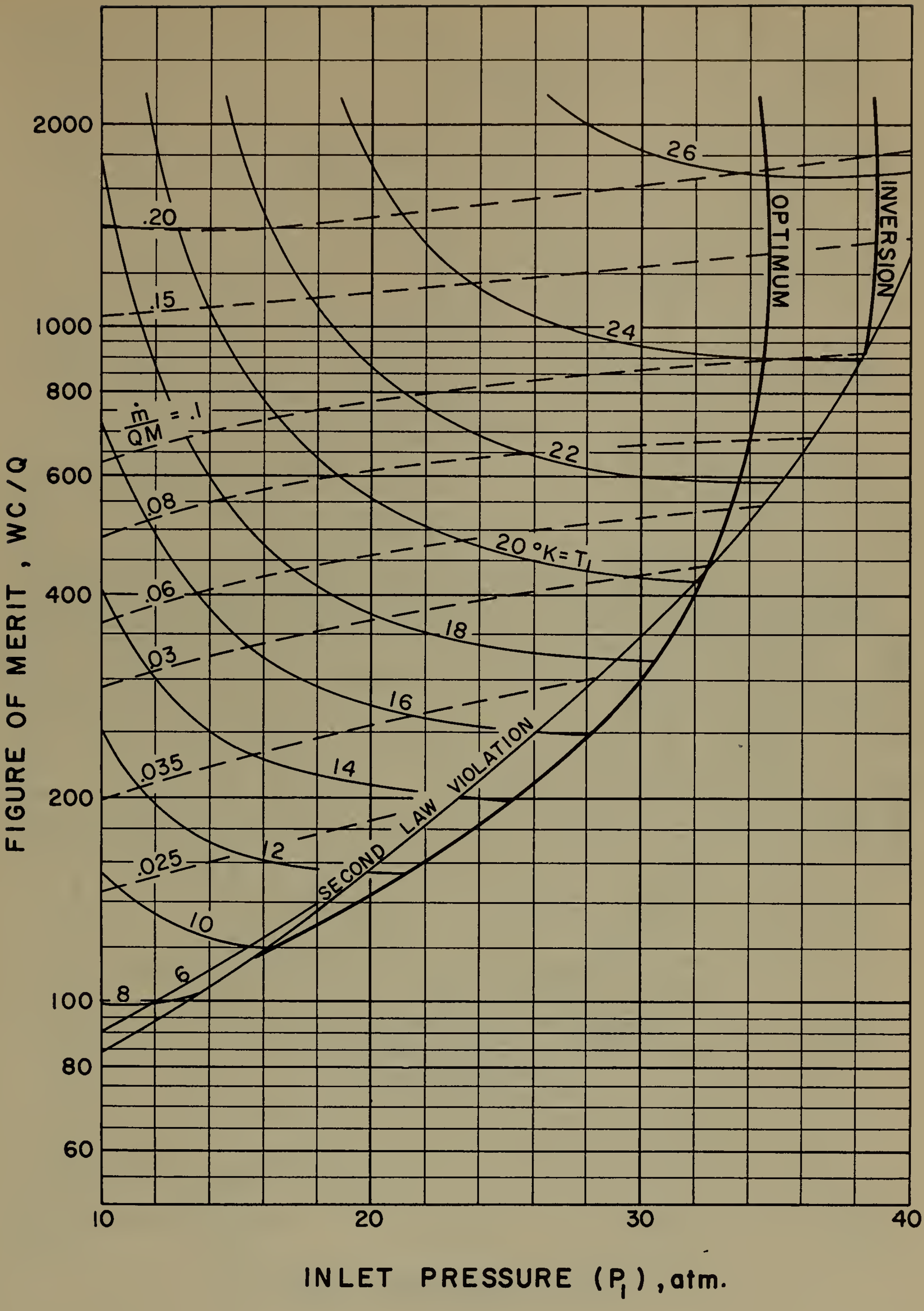

Figure 12. Refrigerator Performance - Helium Gas - $(T_1 - T_5) = 1.0°K$

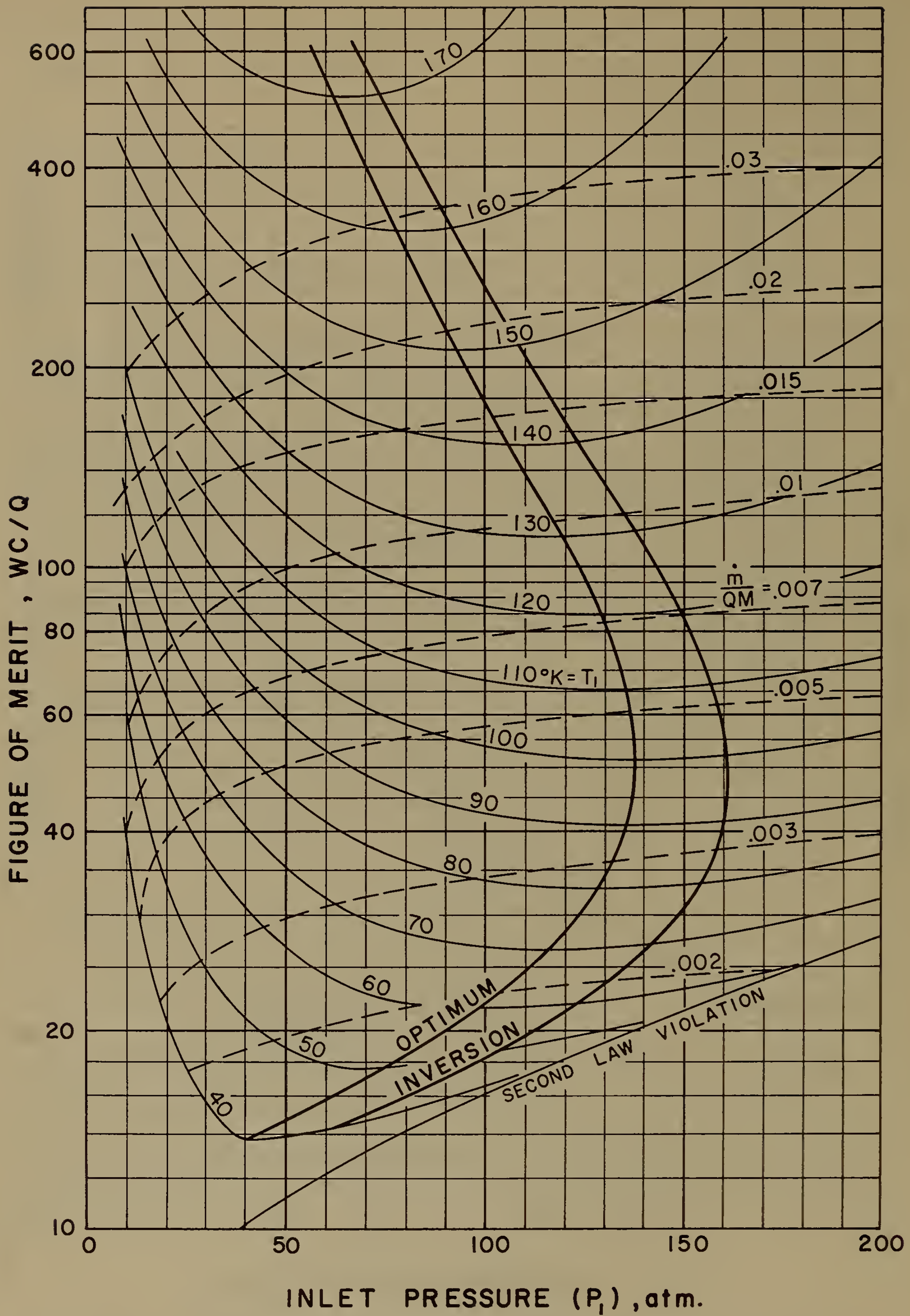

Figure 13. Refrigerator Performance - Para-Hydrogen - (T_1 - T_5) = 0. 0°K

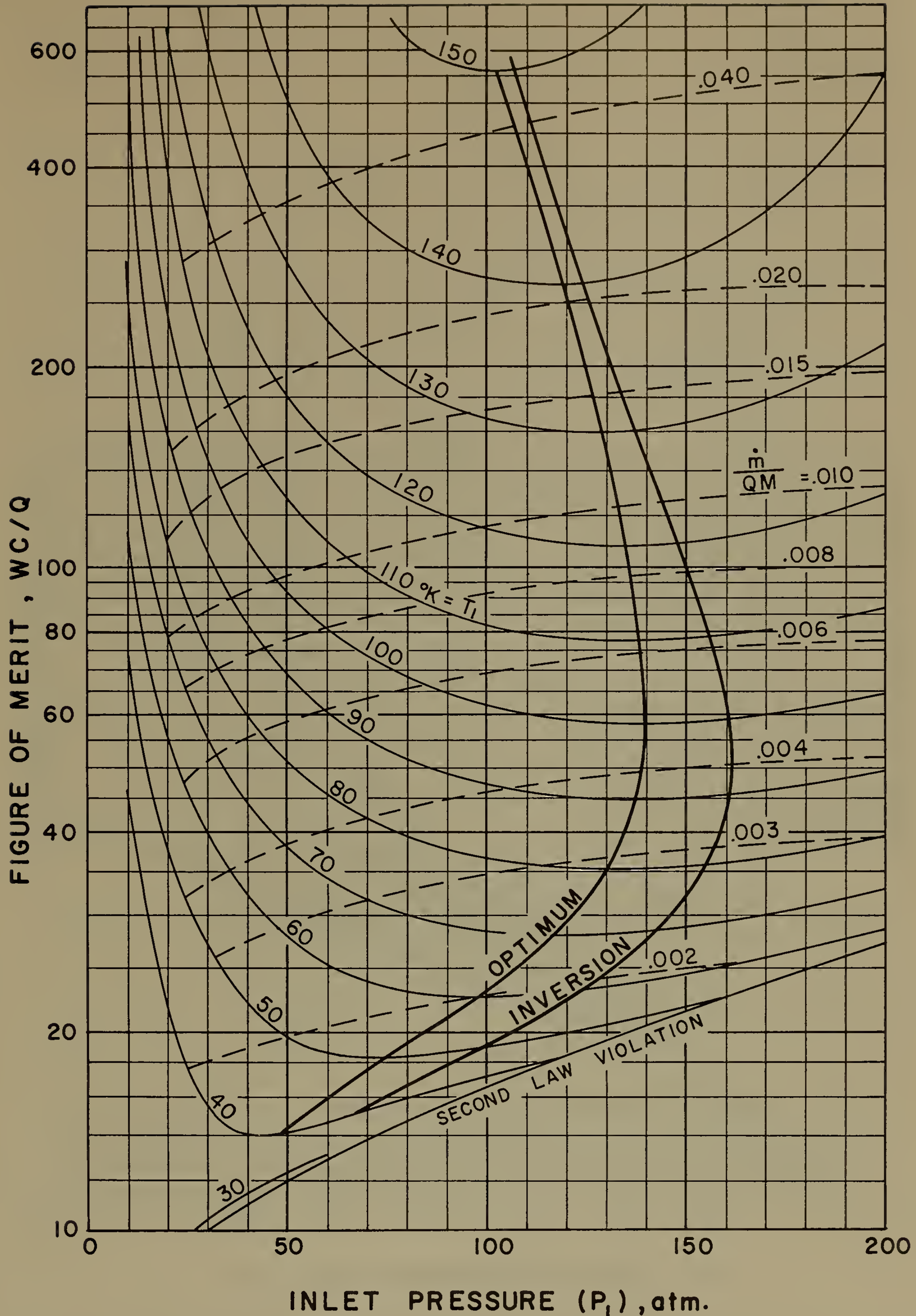

Figure 14. Refrigerator Performance - Para-Hydrogen - $(T_1 - T_5) = 1.0°K$

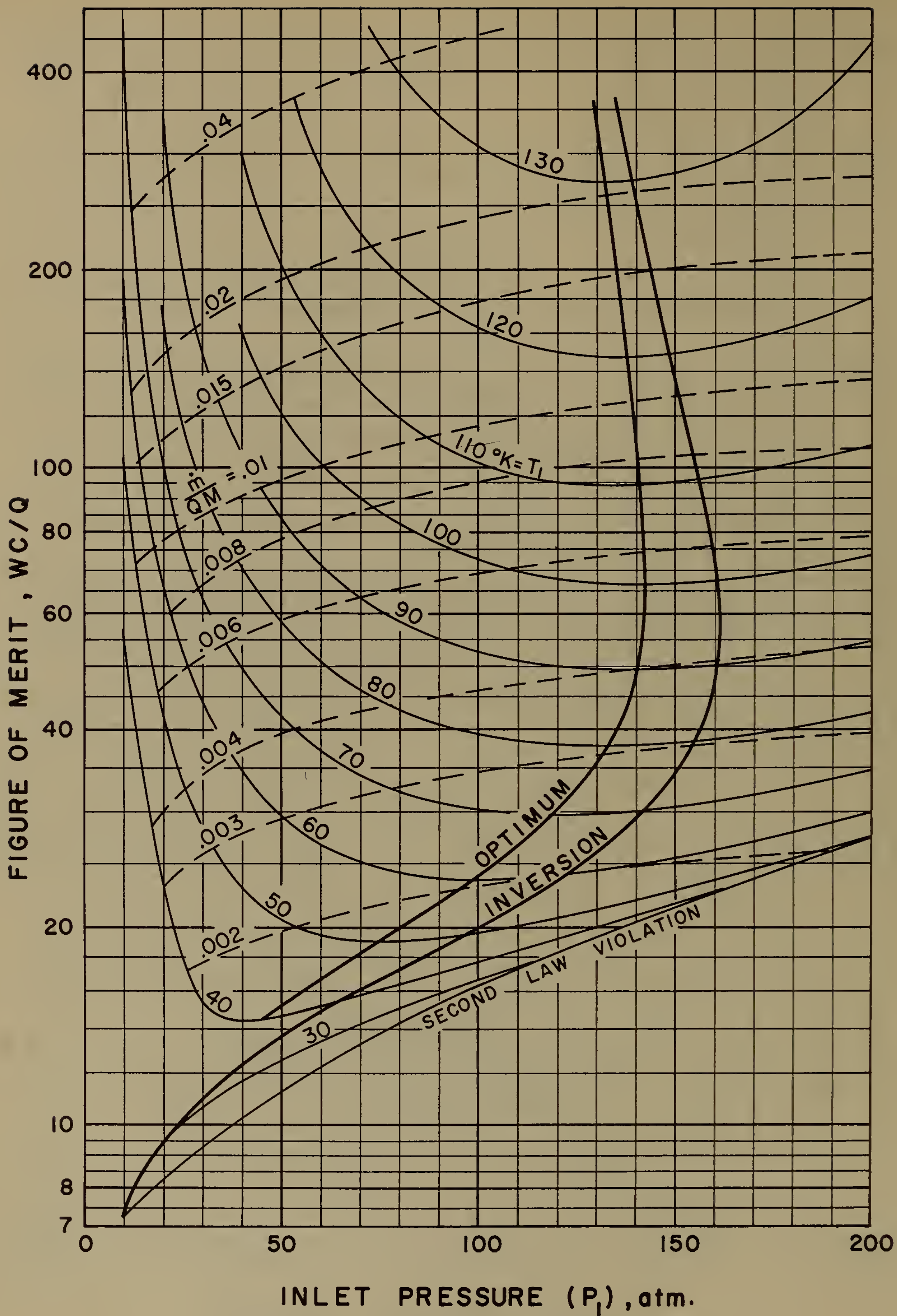

Figure 15. Refrigerator Performance - Para-Hydrogen - (T_1 - T_5) = 2.0°K

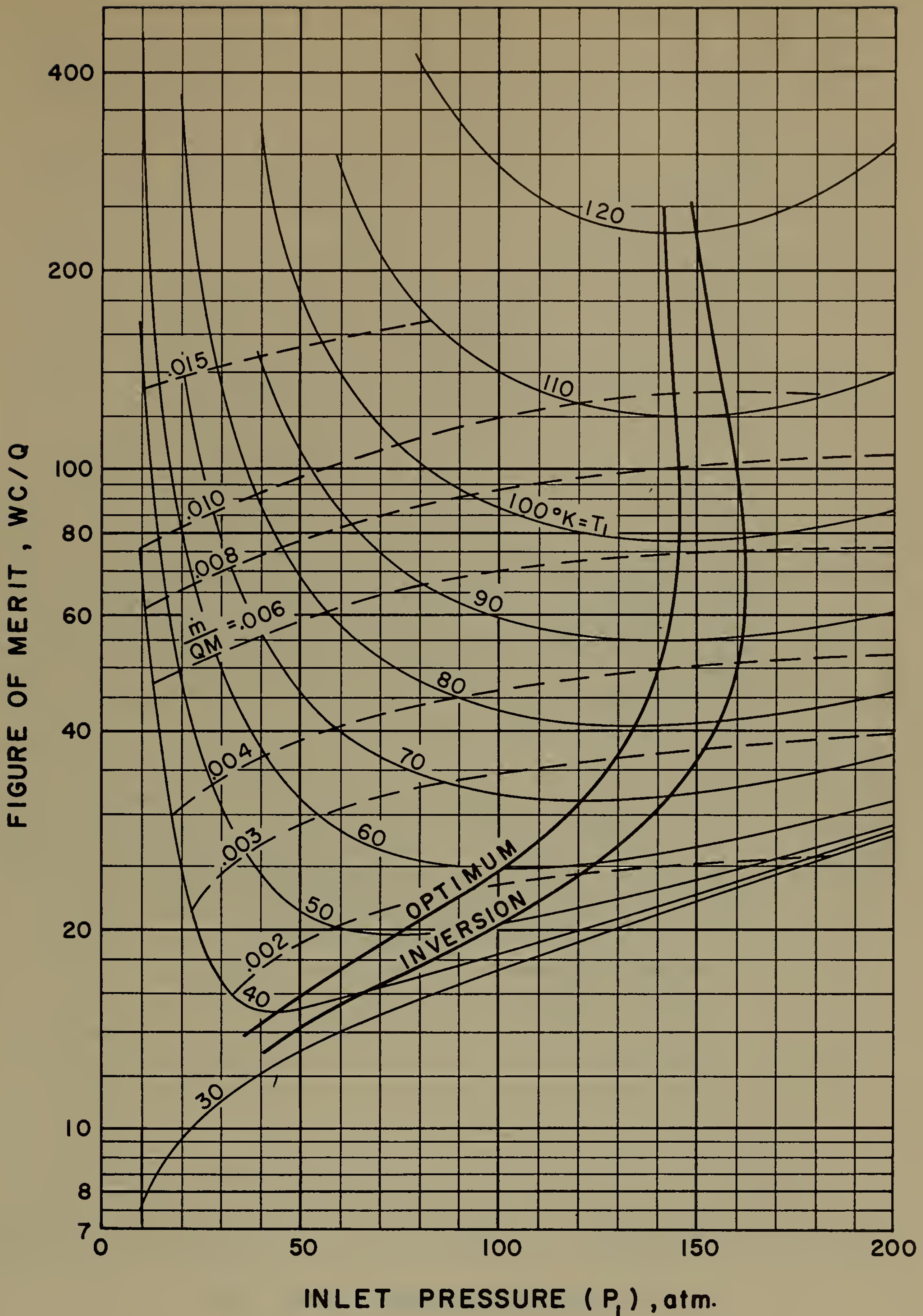

Figure 16. Refrigerator Performance - Para-Hydrogen - $(T_1 - T_5) = 3.0°K$

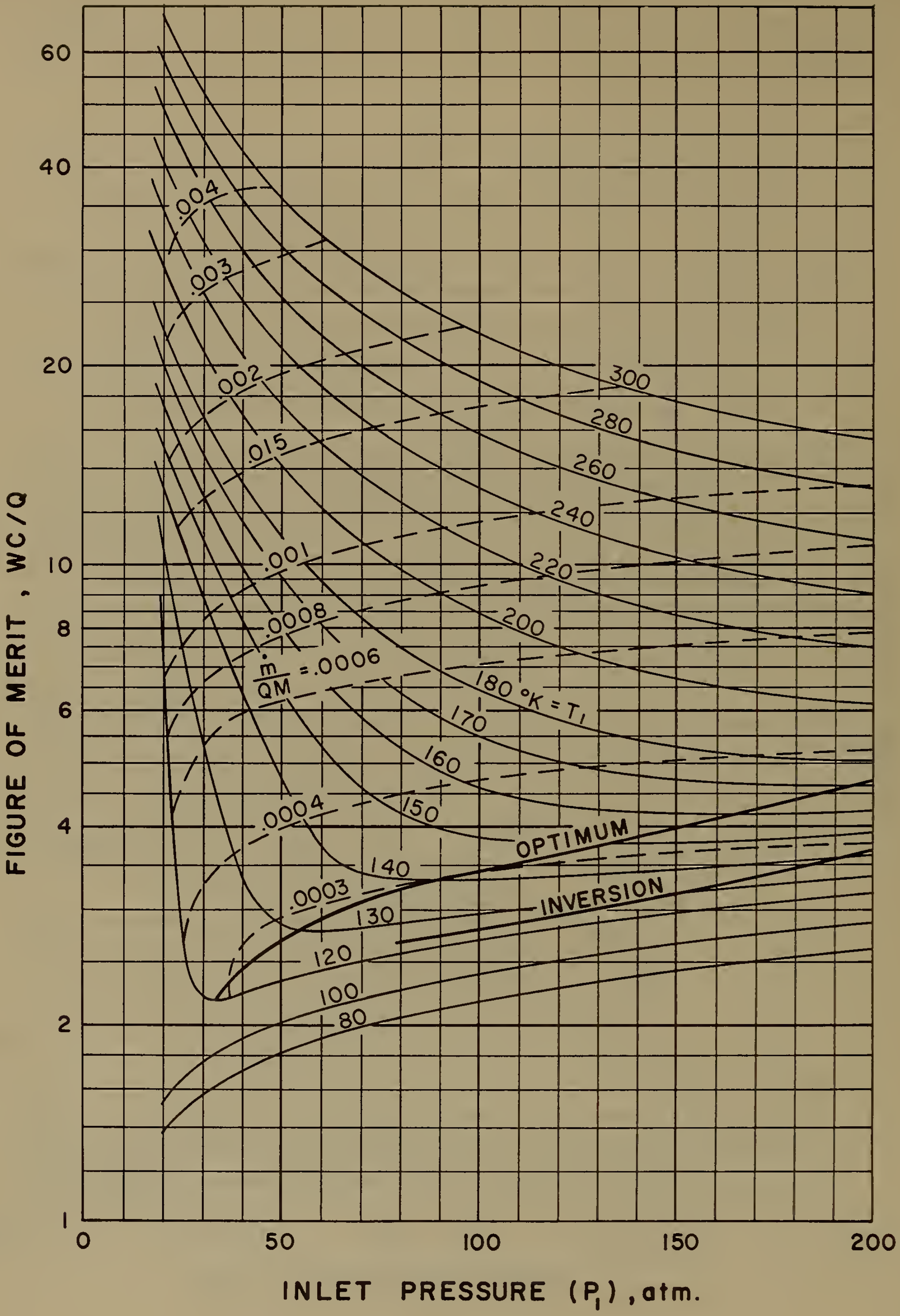

Figure 17. Refrigerator Performance - Nitrogen Gas - $(T_1 - T_5) = 0.0°K$

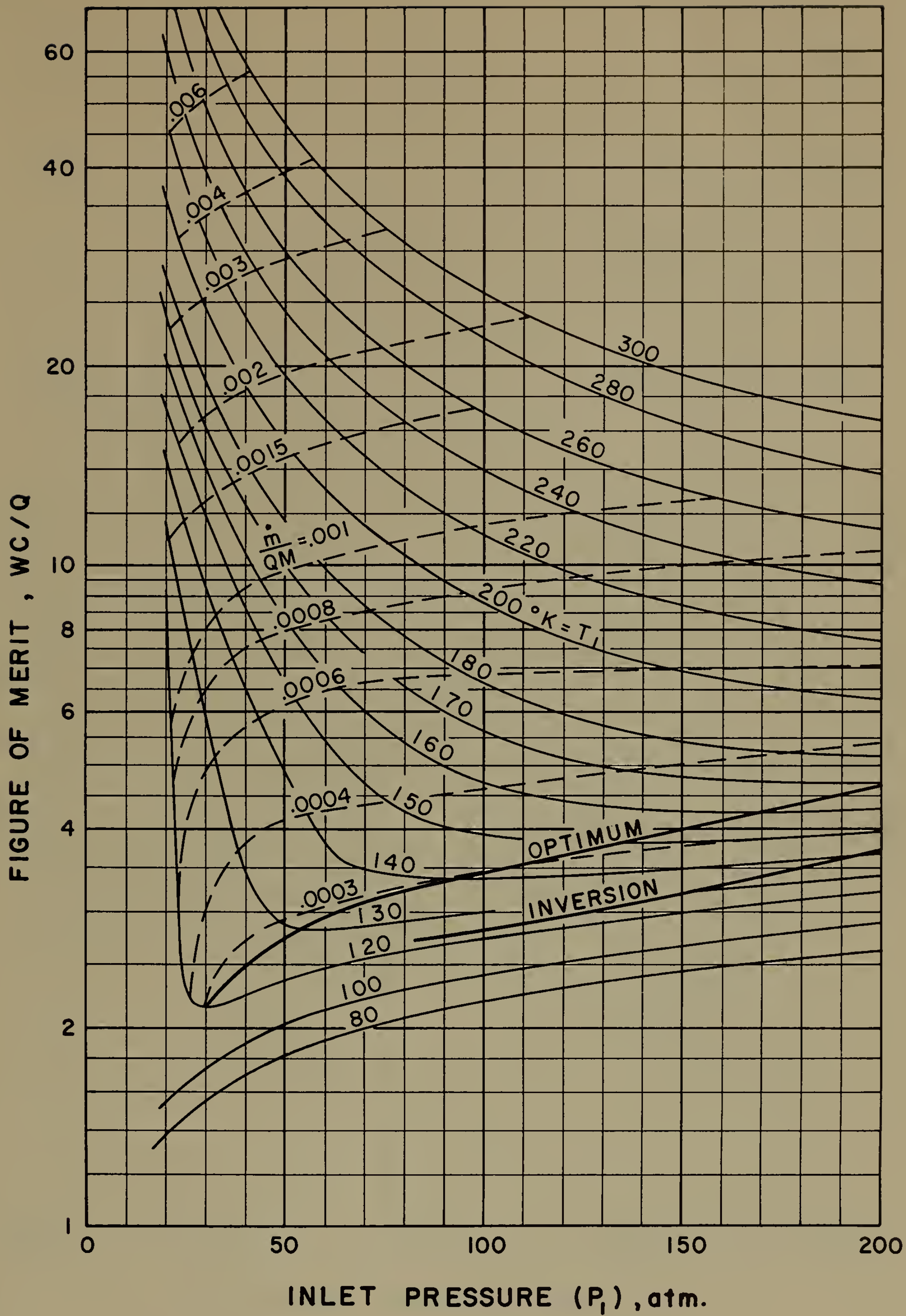

Figure 18. Refrigerator Performance - Nitrogen Gas - $(T_1 - T_5) = 2.0°K$

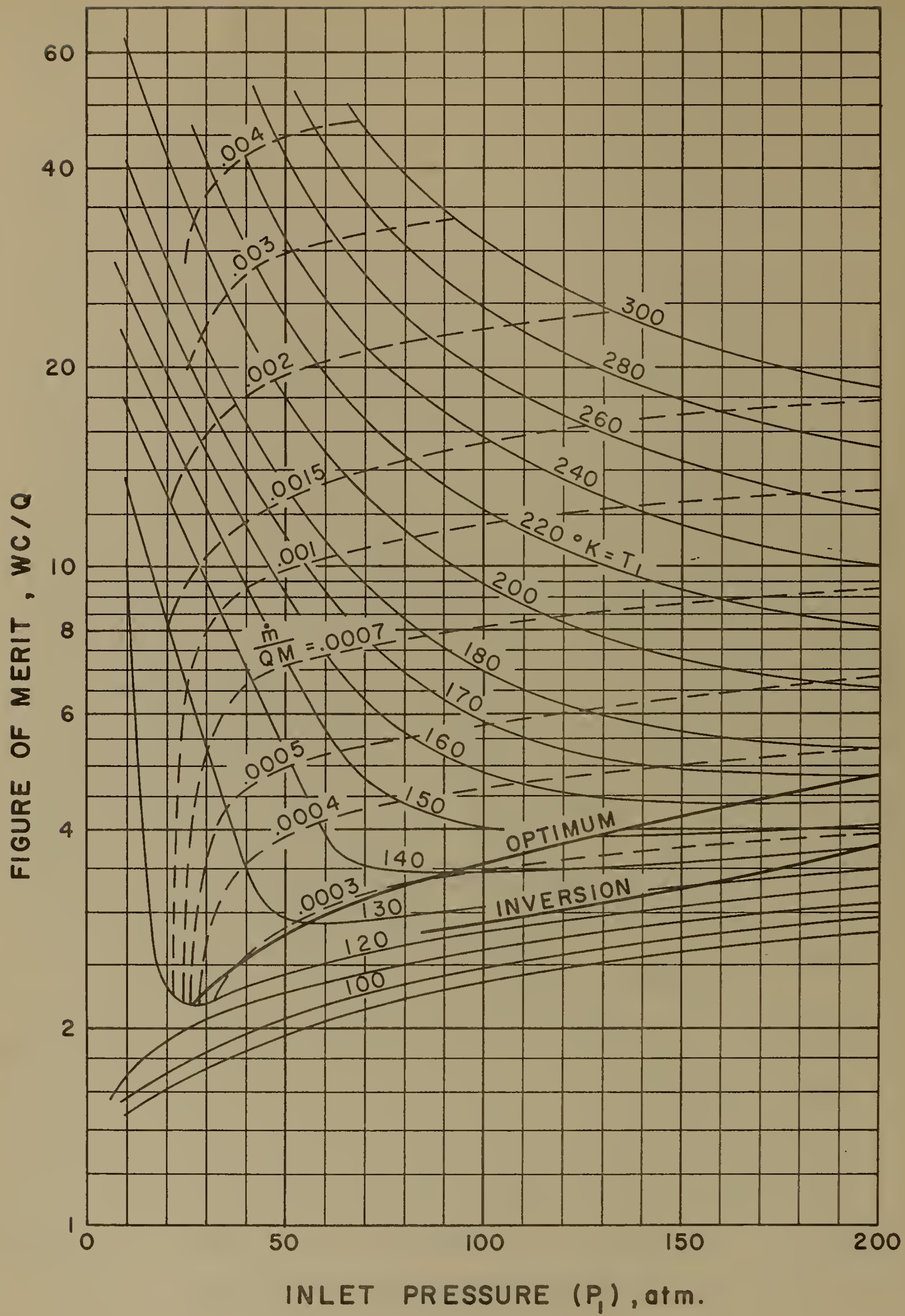

Figure 19. Refrigerator Performance - Nitrogen Gas - $(T_1 - T_5) = 5.0°K$

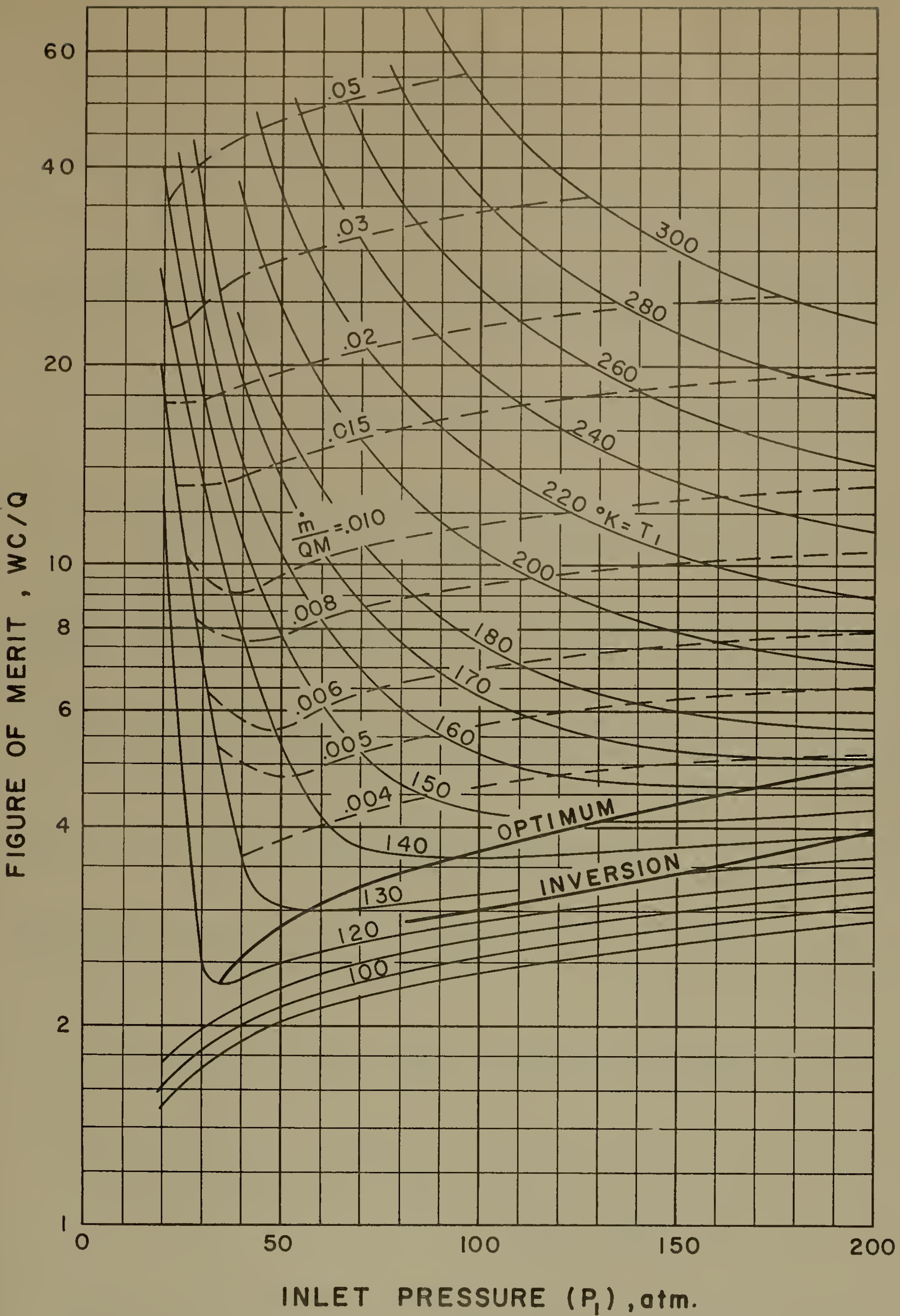

Figure 20. Refrigerator Performance - Nitrogen Gas - (T_1 - T_5) = 10. 0°K

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