WAVE CREST ELEVATION ABOVE THE DESIGN WATER LEVEL DURING SHORE-BREAKING

by

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Bureau of Coastal Data Acquisition
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Florida Department of Natural Resources

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FOREWORD

This work provides a numerical solution for the prediction of wave height behavior relative to the reference still water level during the littoral wave process of shore-breaking. It is basic support methodology required in the development of a multiple shore-breaking wave transformation model described in subsequent work.

The work described herein constitutes partial fulfillment of contractual obligations with the Federal Coastal Zone Management Program (Coastal Zone Management Act of 1972, as amended) through the Florida Office of Coastal Management subject to provisions of contract CM-37 entitled "Engineering Support Enhancement Program". Under provisions of DNR contract C0037, this work was reviewed by the Beaches and Shores Resource Center, Institute of Science and Public Affairs, Florida State University. The document has been adopted as a Beaches and Shores Technical and Design Memorandum in accordance with provisions of Chapter 16B-33, Florida Administrative Code.

At the time of submission for contractural compliance, James H. Balsillie was the contract manager and Administrator of the Analysis/Research Section, Hal N. Bean was Chief of the Bureau of Coastal Data Acquisition, Deborah E. Flack Director of the Division of Beaches and Shores, and Dr. Elton J. Gissendanner the Executive Director of the Florida Department of Natural Resources.

Deborah E. Flack

Deborah E. Flack, Director
Division of Beaches and Shores

November, 1983

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ABSTRACT

Many coastal engineering design solutions regarding wave activity can be accomplished but, only, if the crest elevation of the design wave(s) is known relative to a design water level (DWL). From analysis of field and laboratory data, it is determined that at the shore-breaker position approximately 84% of the wave lies above the DWL. However, the amount of the wave that lies above the DWL during shore-breaking, described by Balsillie (1980, in manuscript) as the alpha wave peaking process, is not constant. Transformation of H'/H, where H is the local wave height and H' is the amount of H lying above the DWL, may be predicted by:

$$\frac{H'}{H} = \frac{H'_b}{H_b} - \Phi_3 \left\{ \tanh \left[\Phi_1 \left(\frac{d}{H} - \frac{d_b}{H_b} \right) \right] \right\}^{\Phi_2}$$

where H_b is the shore-breaking wave height, H'_b is the amount of H_b lying above the DWL, d_b is the water depth at shore-breaking, d is the local water depth, and solutions for Φ_1, Φ_2 and Φ_3 are developed in the text.

INTRODUCTION

Where structures are constructed fronting the shoreline, such as private dwellings, commercial or shore and coastal protection structures, certain types of information are necessary for design purposes. A structure that is exposed or potentially in danger of being exposed to wave action should be designed to withstand the highest design wave expected at the site, if such a design can be economically justified. Such justification will depend critically on the frequency of extreme events, such as wave height and period, and duration of the storm or hurricane waves, the damage potential of the waves,

and the alloted permissible risk. Wave conditions at a coastal locality also depends critically on the water level. Hence, a design still water or mean water level or a range of levels must be established in order to determine the wave forces to which a coastal structure will be subjected. (U. S. Army, 1977).

For example, suppose that the task is assigned to design the elevation of a fishing pier deck where the shore-breaking wave height at the structure from other calculations is estimated to be 4.5 m (approximately 15 feet). If, from theroretical calculations, the sine wave assumption is used (Figure 1) then one-half, or 2.25 m, of the wave will lie above the design water level (DWL). If, however, the Solitary wave assumption is used, the entire 4.5 m wave lies above the DWL. This results in a large design uncertainty of 2.25 m (7.4 feet). While the sine wave assumption may actually be too low to insure a safe deck elevation, the Solitary wave assumption may be in excess, particularly in view of the high costs associated with construction and maintenance in the littoral zone.

The above example though it states the basic problem, is an over-simplification. It is well known that in addition to the design wave crest elevation above the DWL other considerations, in particular the expected horizontal and vertical design wave impact loads, should be applied. The latter is possible only if the nature of wave transformation during shore-breaking is known.

In a previous paper Balsillie (1980) investigated the transformation of waves during the shore-breaking process. It was found that as waves begin to break, the crest height tends to increase reaching a maximum at the shore-breaking position. This phenomenon is termed the alpha wave peaking process. The work was subsequently modified, extended and refined (Balsillie, in manuscript) to account for transformation of the wave height for the entire alpha peaking process. Alpha peaking may be described as analogous

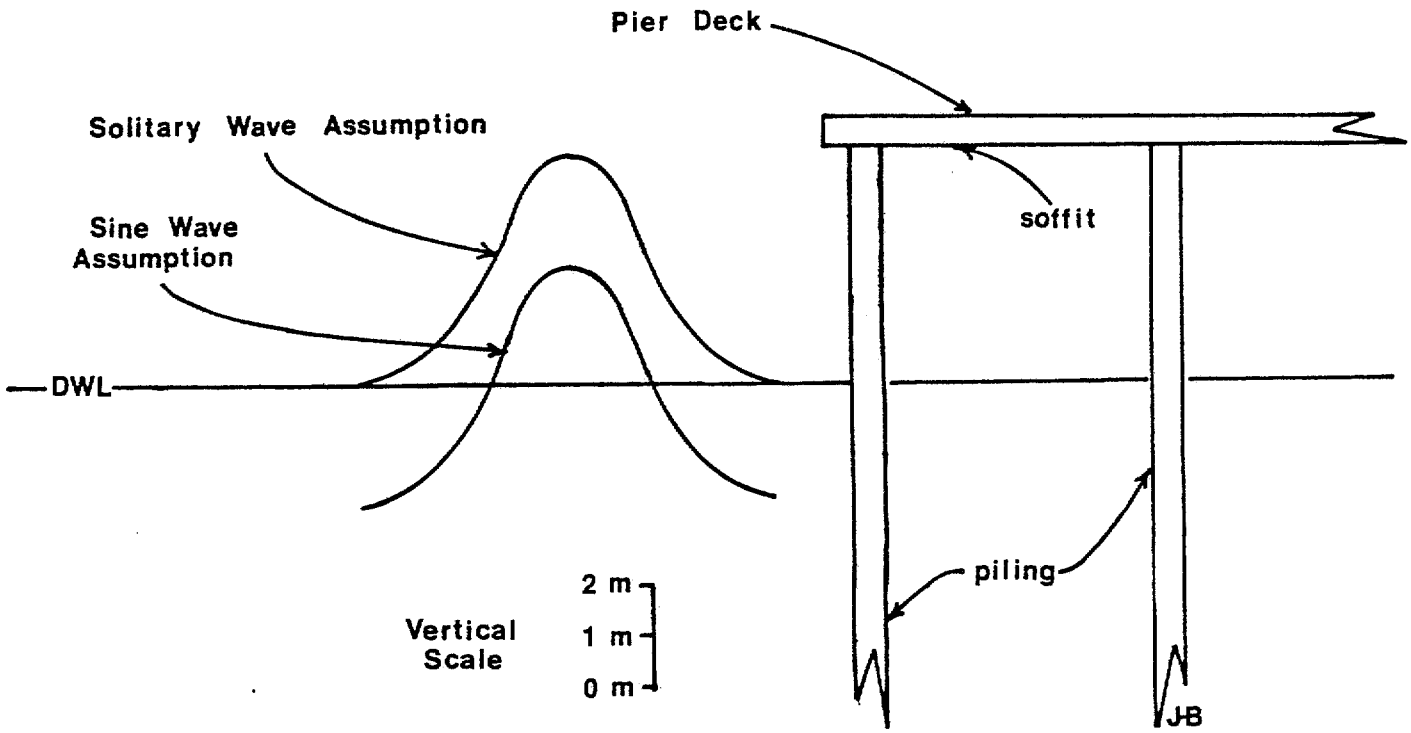


Figure 1. Dependence of design pier deck soffit elevation on wave crest elevation above the design water level (DWL) for two commonly applied theoretical approaches (after C. J. Galvin, personal communication, 1978).

to refraction mechanics in deeper water except that the wave height tends to increase. At the same time, the amount of the wave crest lying above the DWL tends to increase, again, apparently reaching a maximum at the shore-breaking position. Since the amount of the wave that lies above the DWL is proportional to the potential energy of the wave, it becomes important to consider in applications dealing with design solutions for shore-breaking wave mechanics. It is this latter phenomenon which is investigated in this paper.

PREVIOUS WORK AND ATTEMPT AT DUPLICATION

An estimation of the amount of the local wave crest that lies above the DWL, H', can be attempted using various wave theories. However, the applicability of classical theories, although they have demonstrated relevance for predicting "deeper water" wave conditions, are not specifically designed to predict wave behavior during the shore-breaking process, particularly since shore-breaking waves are not symmetrical in profile view. Rather, the crests become progressively asymmetrical and distorted.

Despite the underlying importance of the issue considered, surprisingly little work has been produced which addresses the phenomenon. In fact, of the work done, the paper of Bretschneider (1960), that by its singularity, becomes a classical accomplishment. Bretschneider's results provide for a measure of H' relative to the still water level (i.e., SWL which in this paper represents the DWL) as illustrated in Figure 2. However, for the entire range of conditions represented by Figure 2, no mathematical description has been developed. One can, however, simplify conditions where deep water (i.e., $d/L > 0.5$ or $d/(g\ T^2) > 0.08$) and shallower water (i.e., $d/L < 0.5$ or $d/(g\ T^2) < 0.08$) regions are treated separately (where d is the local water depth, g is the acceleration of gravity, and T is the wave period).

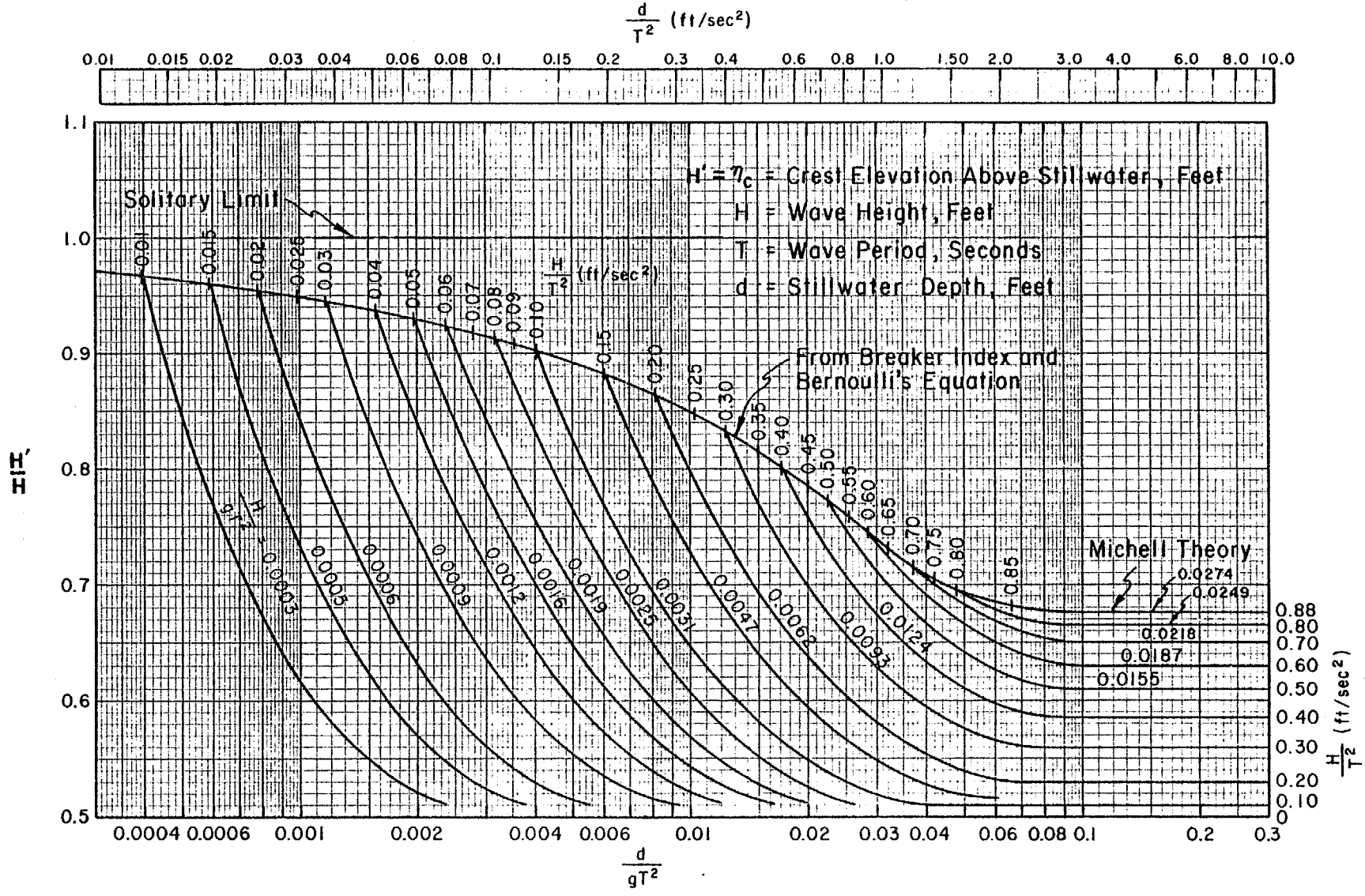


Figure 2. Ratio of wave crest height above the still water level to the wave height (from U. S. Army, 1977; originally from Bretschneider, 1960).

In deep water the depth is great enough that the relative parameters, d/L and $d/(g\ T^2)$, are affected only by the driving wind forces generating the waves and modification by dispersion mechanics, not by the bathymetry. For these conditions, Gaillard (1904) proposed that:

$$\frac{H_o'}{H_o} = 0.5 + \frac{\pi\ H_o}{4\ L_o} \tag{1}$$

where H_o is the deep water wave height, H_o' is that portion of the deep water wave height lying above the DWL, and L_o is the deep water wave length. When small amplitude (Airy) wave theory is applied, $L_o = g\ T^2/(2\ \pi)$, equation (1) becomes:

$$\frac{H_o'}{H_o} = 0.5 + \frac{\pi^2\ H_o}{2\ g\ T^2} \tag{2}$$

where Gaillard's development is based on trochoidal wave theory.

The Levi-Civita (1924) development based on a fifth order Stokes-Levi-Civita solution provides that:

$$\frac{H_o'}{H_o} = 0.5 + \frac{\pi^2\ H_o}{2\ g\ T^2} + \frac{1}{3}\left(\frac{\pi^2\ H_o}{L_o}\right)^3 \tag{3}$$

that when, as for equation (2), Airy wave theory is used becomes:

$$\frac{H_o'}{H_o} = 0.5 + \frac{\pi^2\ H_o}{2\ g\ T^2} + \frac{1}{3}\left(\frac{2\ \pi^2\ H_o}{g\ T^2}\right)^3 \tag{4}$$

In addition to the above developments, it becomes useful to know the maximum value of H_o'/H_o that may be attained. The maximum deep water wave steepness is given by Mitchell (1893) as:

$$\left(\frac{H_o}{L_o}\right)_{max} = \frac{1}{7} \tag{5}$$

or where $L_o = g\ T^2/(2\ \pi)$ by:

$$\left(\frac{H_o}{g\ T^2}\right)_{max} = \frac{1}{14\ \pi} \tag{6}$$

The preceding equations are evaluated in Figure 3 using data reported by the Beach Erosion Board (1941). Combination of these equations yields the result that the maximum value of H_o'/H_o is on the order of from 0.61 to 0.64.

In shallower water (i.e., transistional and shallow water depths) Bretschneider's (1960) nomograph may be approximately constructed according to the following treatment. The general equation is given by:

$$\frac{H'}{H} = 0.4525 + 1.909\ \Phi_1\ \Phi_2 \quad \left\| \begin{array}{l} \frac{H'}{H} \geq 0.5 \\ \frac{d}{H} \leq 1.28 \end{array} \right. \tag{7}$$

in which H is the local wave height. The evaluation of the coefficient Φ_1 is based on the Second Order Stokes wave theory, given by:

$$\frac{H'}{H} = 0.5 + \frac{\pi}{8}\ \frac{H}{L}\ \frac{\left(\cosh \frac{2\ \pi\ d}{L}\right)\left(2 + \cosh \frac{2\ \pi\ d}{L}\right)}{\sinh \frac{2\ \pi\ d}{L}} \tag{8}$$

where L is the local wave length.

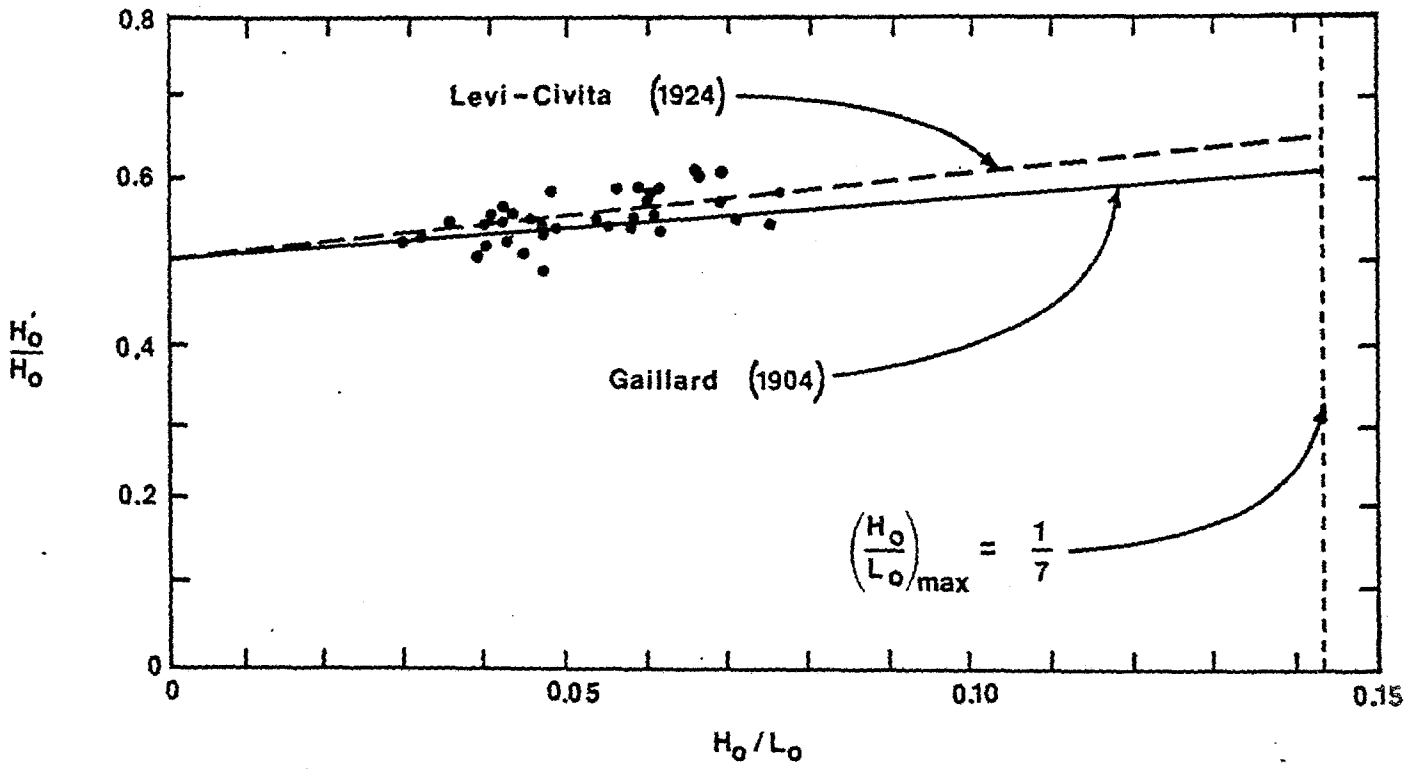


Figure 3. Percentage of the deep water wave height above the still water level as a function of the deep water wave steepness.

It is characteristic of most classical wave theories that the local wave length must be known when, in fact, it is seldom known in task-oriented coastal engineering applications. For this reason, L in equation (8) is evaluated by $L = T\sqrt{g\,d}$, assuming that the approximation holds even in the near shallow water regions of the transitional water depth region, to yield the modification:

$$\Phi_1 = \left[\frac{\pi}{8}\,\frac{H^2}{g\,T^2}\,\frac{\left(\cosh 2\pi\sqrt{\frac{d}{g\,T^2}}\right)\left(2+\cosh 2\pi\sqrt{\frac{d}{g\,T^2}}\right)}{\left(\sinh 2\pi\sqrt{\frac{d}{g\,T^2}}\right)^3}\right]^{0.5} \tag{9}$$

An additional correction factor, Φ_2, appears to be required which may be approximated by:

$$\Phi_2 = 3.089\left(\frac{H}{g\,T^2}\right)^{0.4446} \tag{10}$$

Equation (7) is tested using prototype laboratory wave and field hurricane waves from Bretschneider (1960). As illustrated in Figures 4 and 5, the agreement appears good.

The maximum value of H'/H was derived by Bretschneider (1960) using the breaker index and Bernoulli equation. An approximation for transistional and shallow water depths may be given by:

$$\frac{H'}{H} = 0.963 - 20.51\,\frac{d}{g\,T^2} + 31.02\left(\frac{d}{g\,T^2}\right)^{1.25} \tag{11}$$

Equations (7) and (11) are plotted in Figure 6 in the attempt to reconstruct Bretschneider's nomograph of Figure 1, for transitional and shallow water depths only.

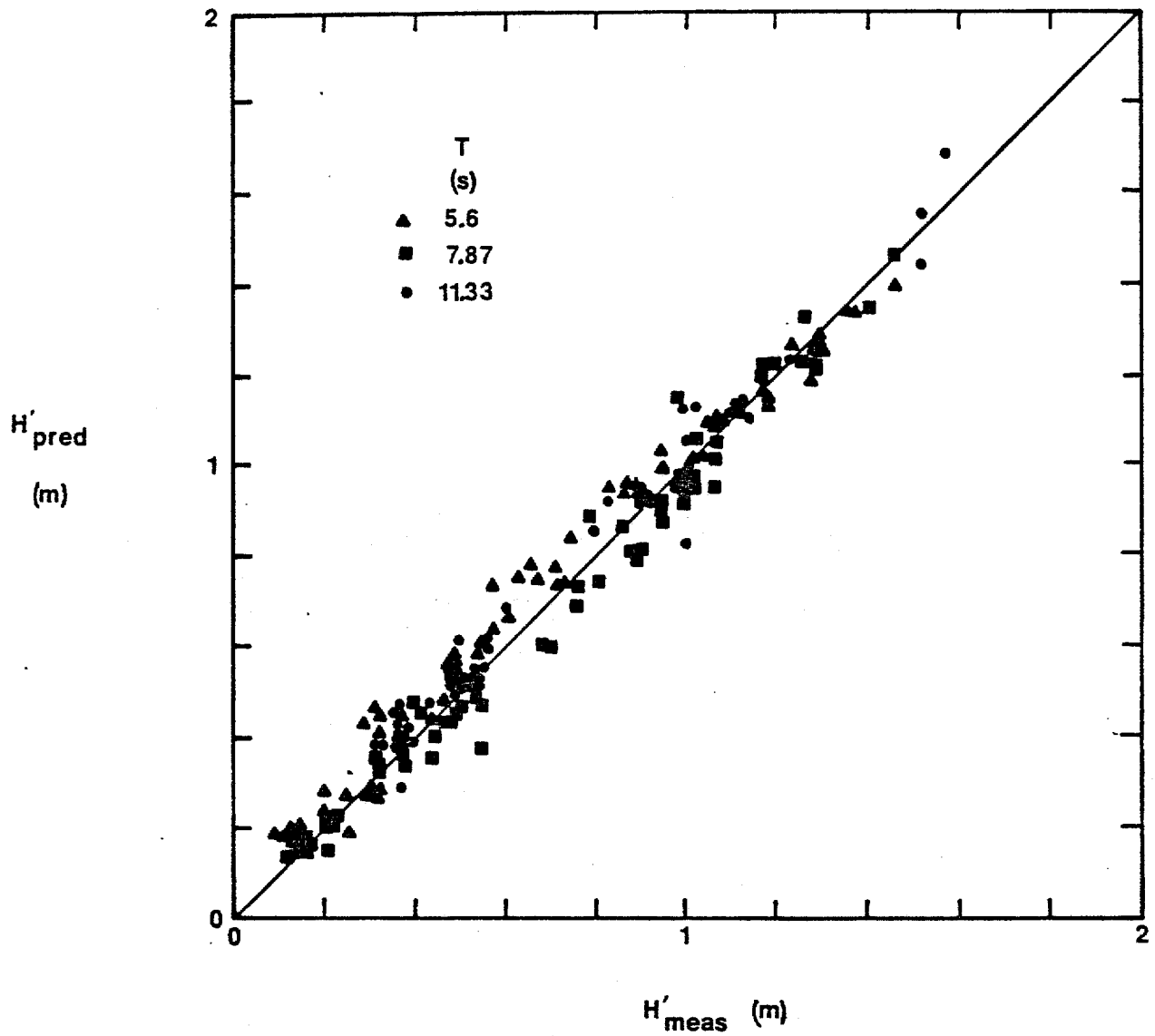


Figure 4. Predicted H' from equation (7) versus prototype Beach Erosion Board wave tank data reported by Bretschneider (1960).

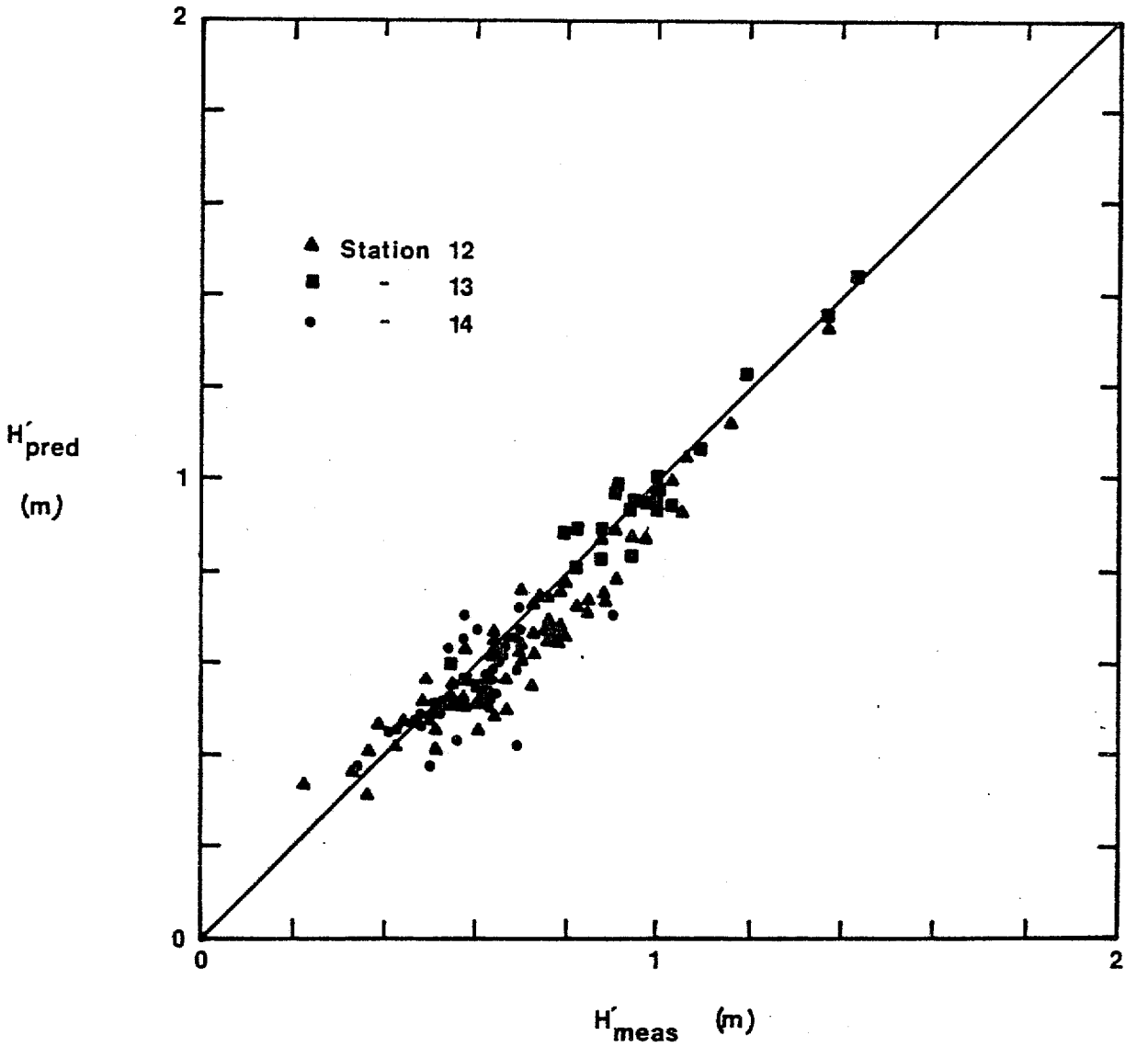


Figure 5. Predicted H' from equation (7) versus Lake Okeechobee hurricane wave data reported by Bretschneider (1960).

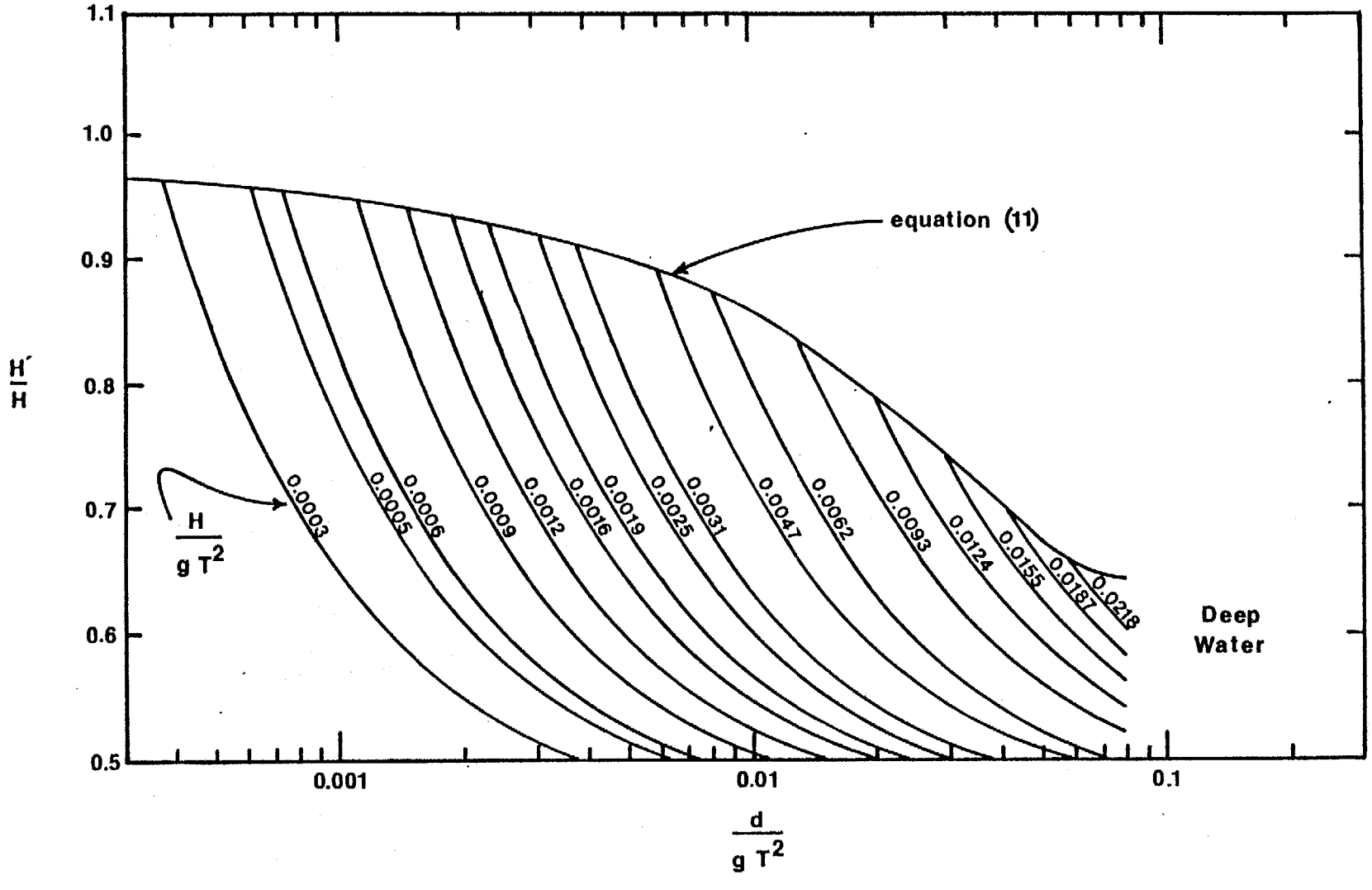


Figure 6. Ratio of crest elevation above the still water level to wave height from equations (7) and (11).

RE-ANALYSIS

The above treatment for shallower water depths pertains to wave conditions at a specified or constant depth. It would, in addition, be useful to know how the value of H'/H behaves as the wave progresses across a shoaling bathymetry as the wave nears shore-breaking. The data of Putnam (1945) are used to assess equation (7). These data and results from equation (7) are plotted in Figure 7 (equation (7) shown as the dashed curves). The plots of Figure 7 illustrate that as alpha wave peaking progresses, the curve from equation (7) is quite gentle compared to the trend exhibited by the data. There would, therefore, appear to be a need to reasses the predicting expression. Resolution, or at least some enlightenment, may be accomplished by identifying the limiting boundary conditions of the shore-breaking process.

Terminal Boundary Conditions

The first boundary condition occurs where the value of H'/H is evaluated at the shore-breaking position, that is, the value of H_b'/H_b. Using available field and laboratory data, the depth of water at shore-breaking, d_b (referenced in this work to the SWL), and the wave height at shore-breaking, H_b (see definition sketch of Figure 8) may be related according to:

$$\frac{d_b}{H_b} = 1.28 \qquad (12)$$

illustrated in Figure 9. The data of Figure 9 are discussed by Balsillie (in manuscript) and include 157 field measurements and 251 laboratory results. Equation (12) is also that proposed by McCowan (1894). The field data of Weishar (1976) are not included in the analysis leading to equation (12), but results in an average value of $d_b/H_b = 1.27$ for 116 sampled waves, which is in close agreement with equation (12).

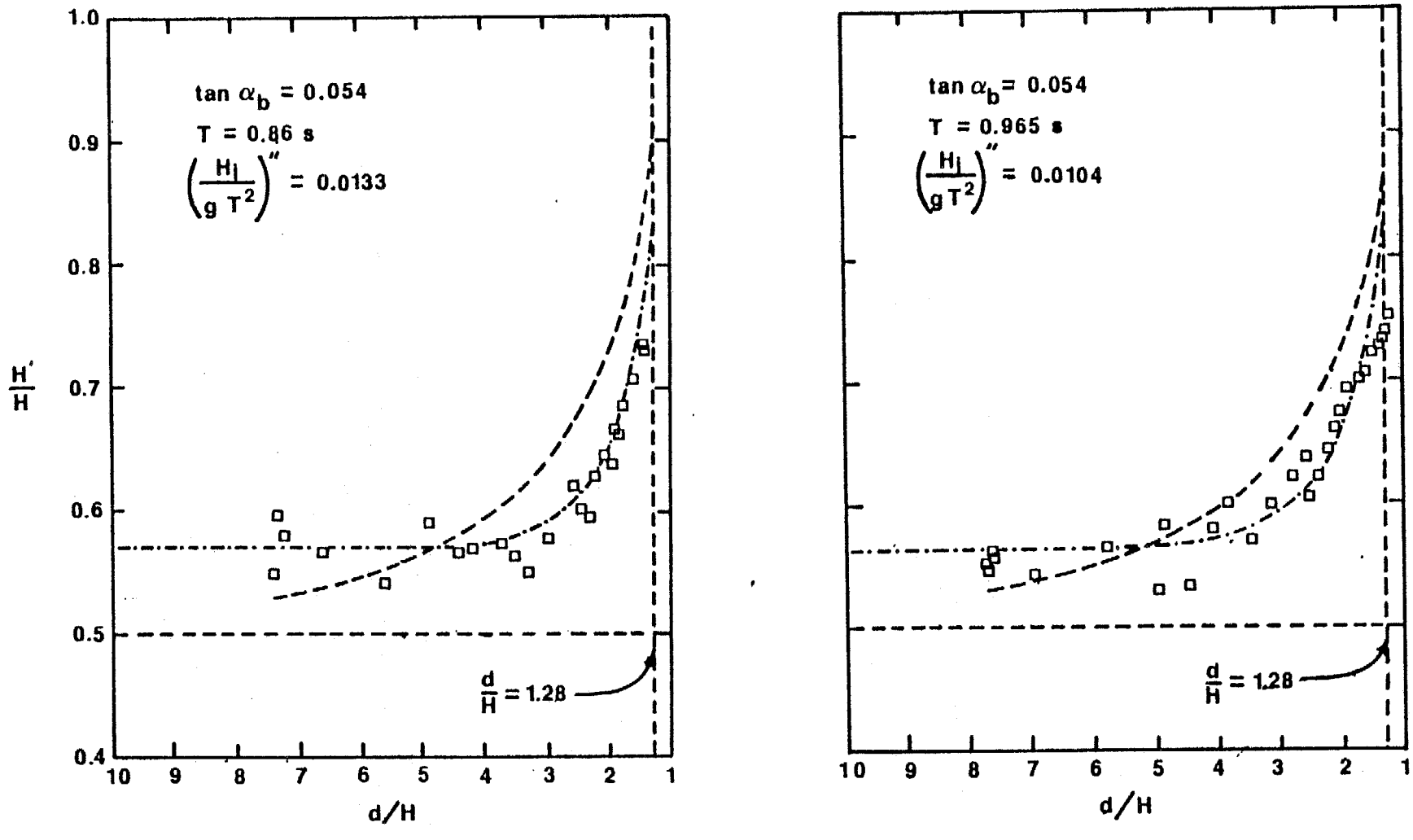


Figure 7. Transformation of H'/H during shore-breaking; dashed curves from equation (7), dash-dot-dash curves from equation (27); data from Putnam (1945).

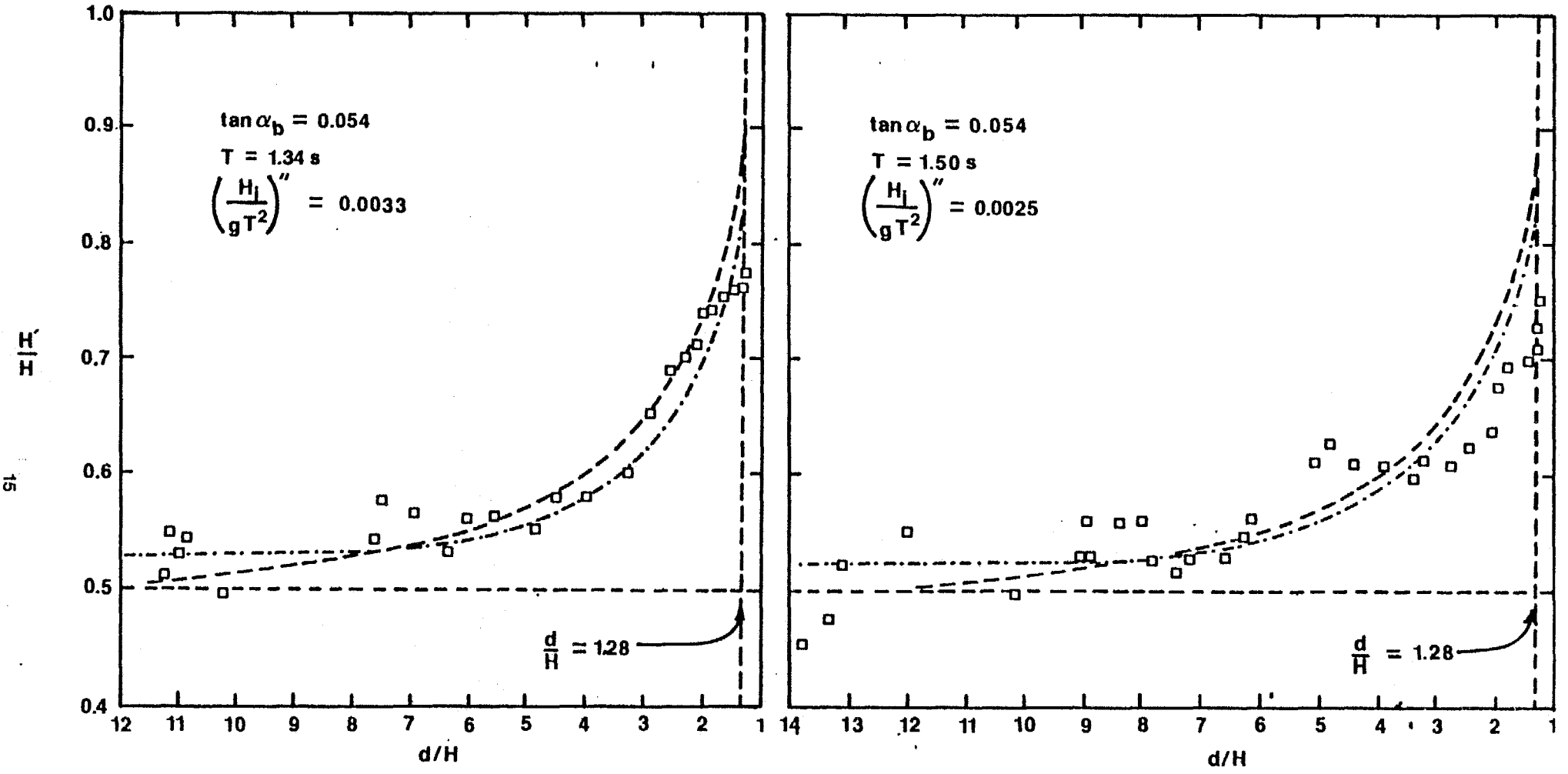
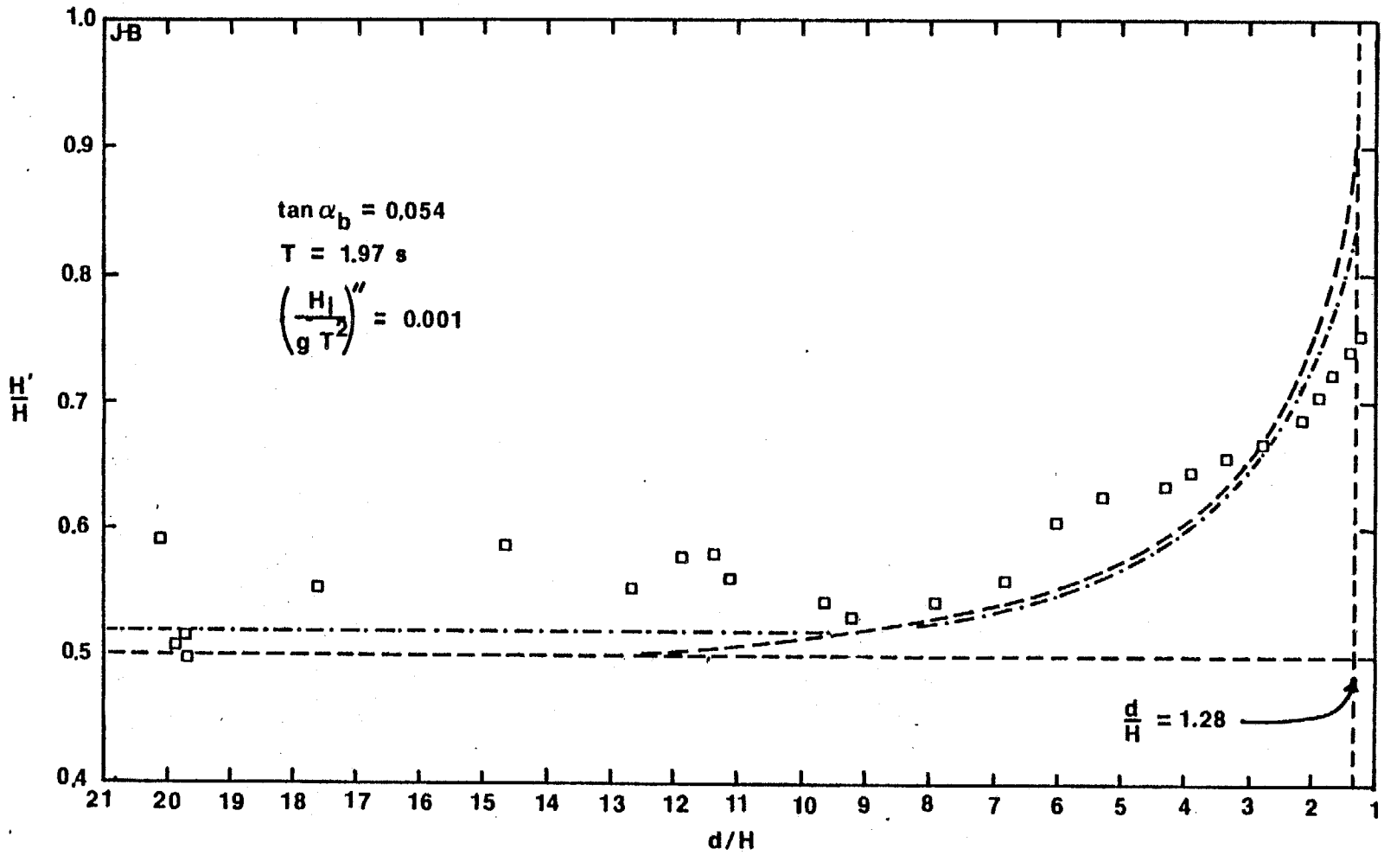


Figure 7. (cont.)



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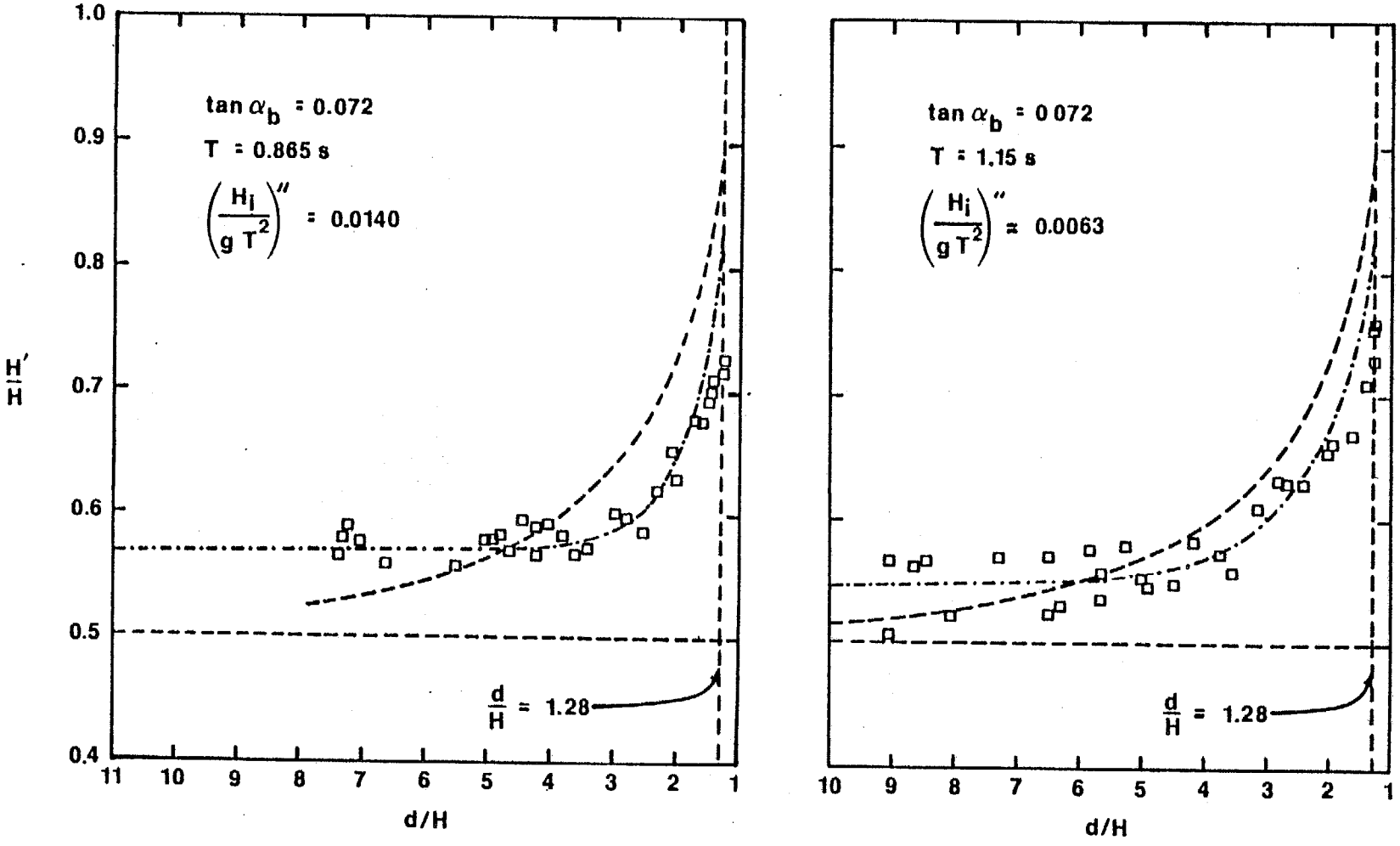

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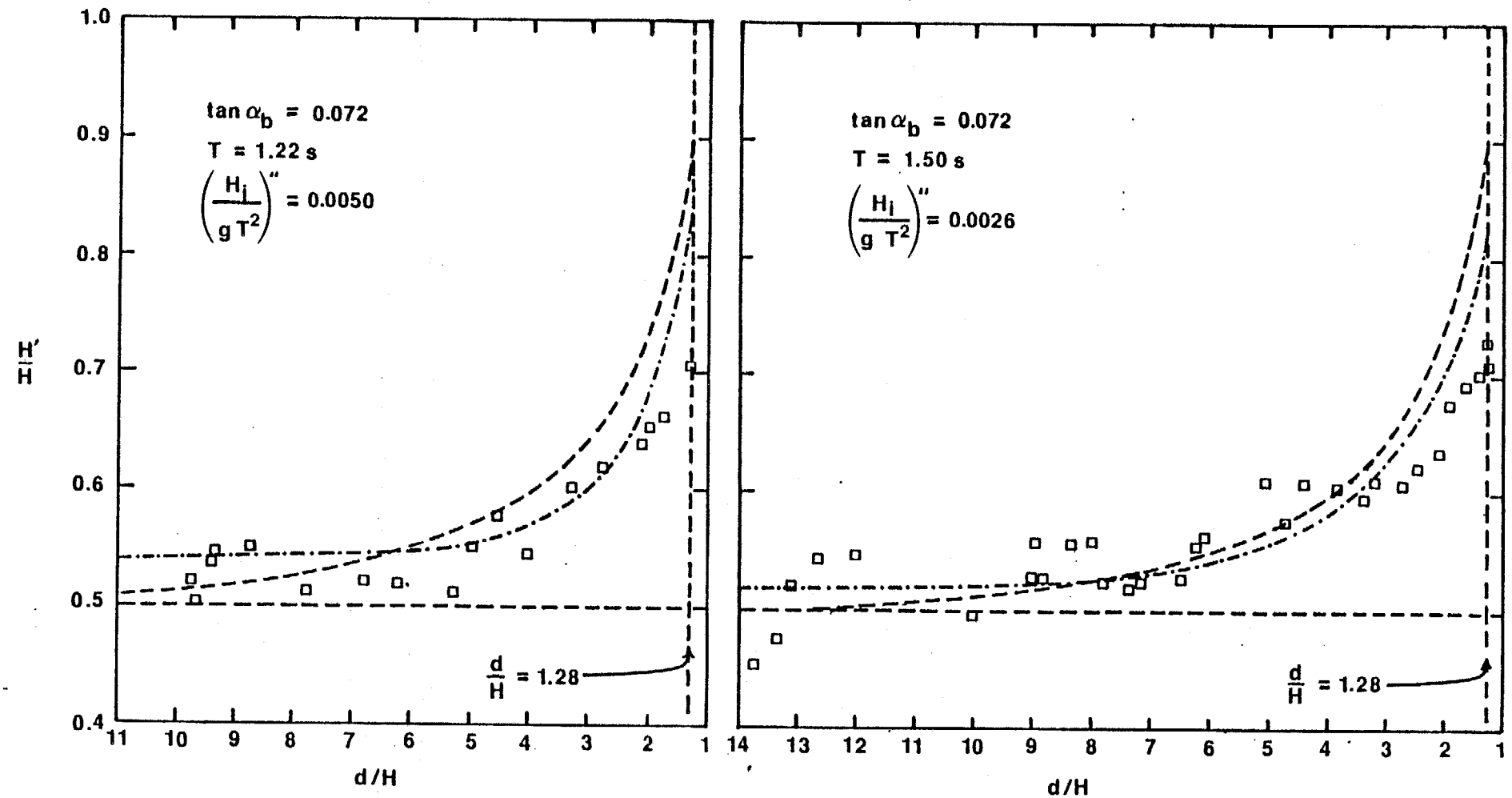

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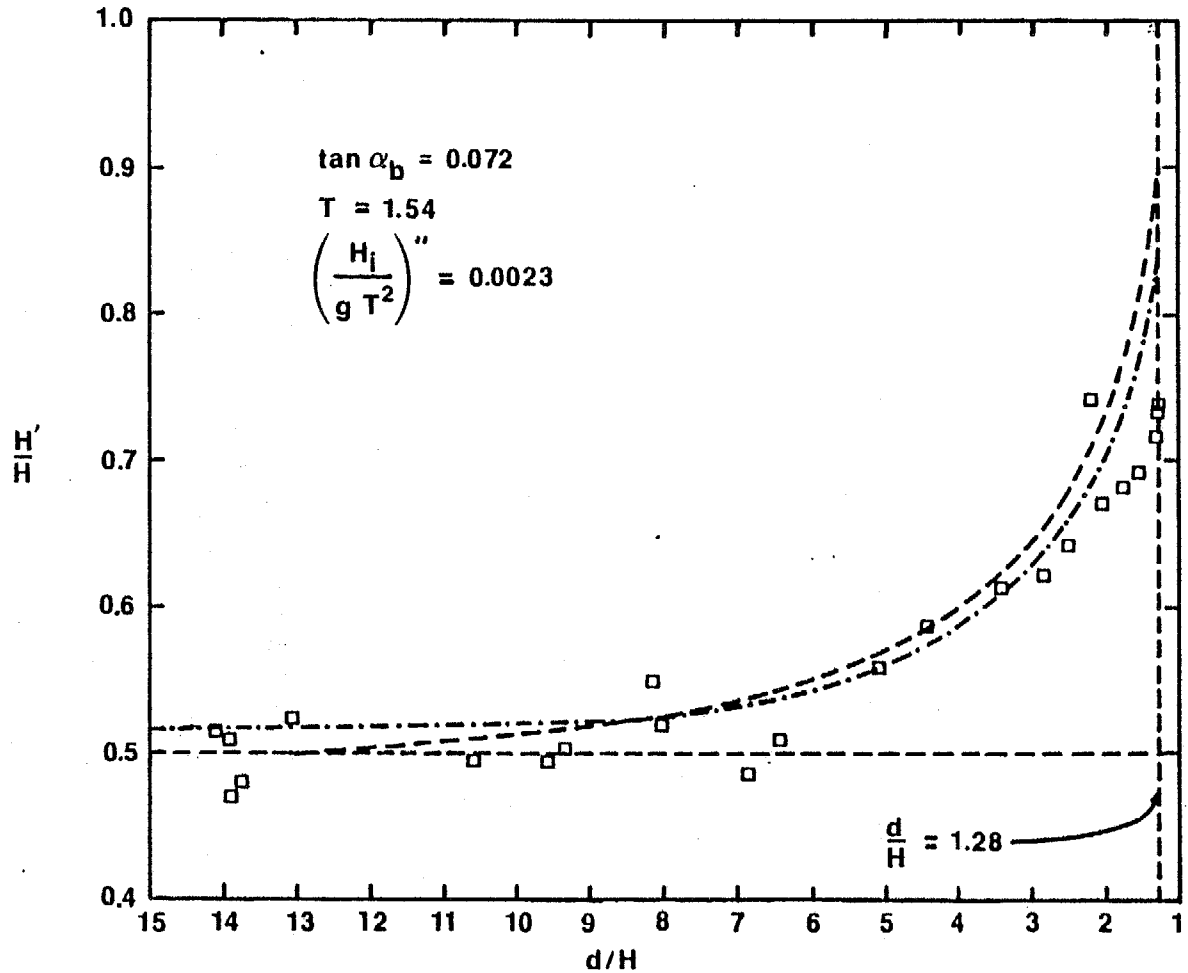

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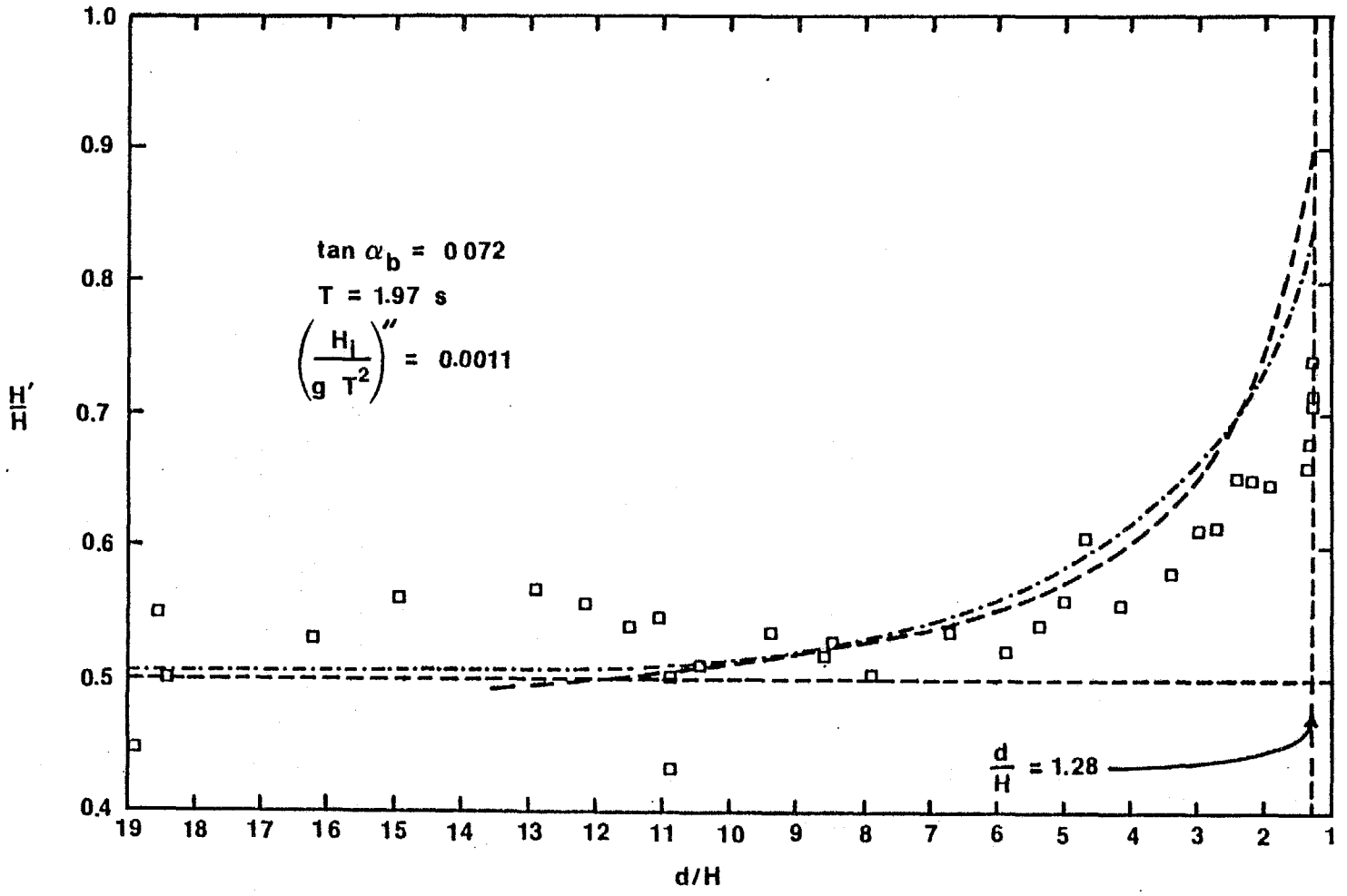

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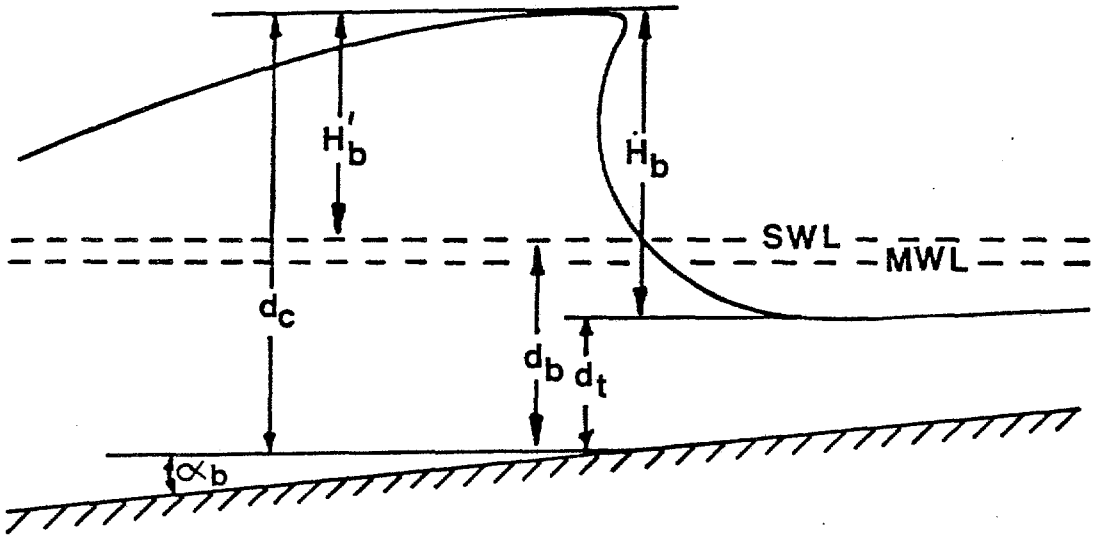

Figure 8. Definition sketch for variables describing the wave at the shore-breaking position (plunging-type shore-breaker).

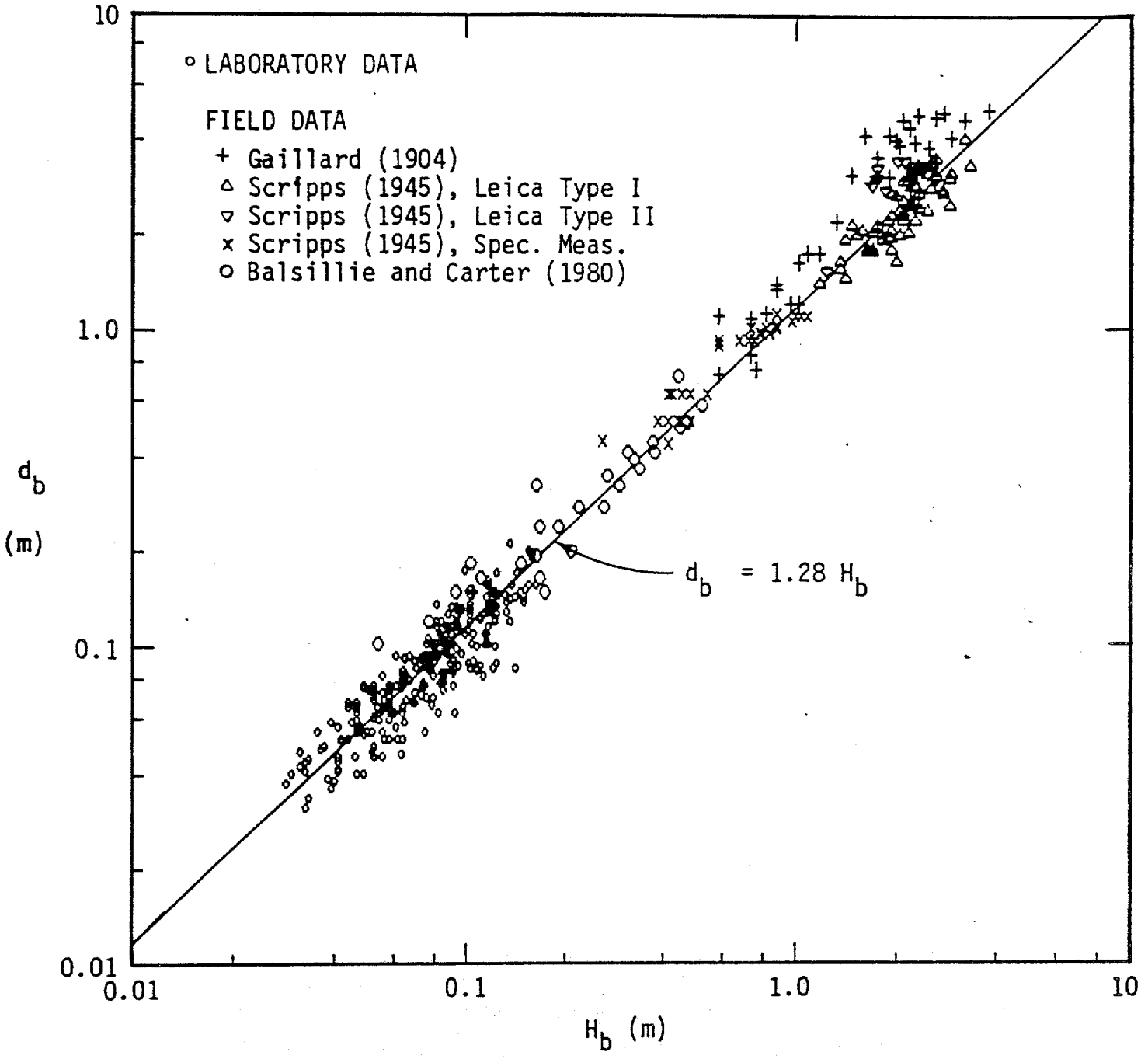

Figure 9. Relationship between the water depth at shore-breaking, d_b, and the shore-breaking wave height, H_b (from Balsillie, in manuscript).

Using the laboratory data of Iverson (1952) and field data of Balsillie and Carter (1980) and Balsillie (1980), the relationship between the shore-breaking wave height and the wave trough depth, d_t (d_t is the vertical distance from the wave trough located just shoreward of the breaking wave crest to the bottom, referenced to the SWL), may be found. These data are plotted in Figure 10, and indicate that $d_t/H_b = 1.092$ for 88 wave samples. Weishar (1976) reports that $d_t/H_b = 1.124$ for 116 field measurements. A weighted average from the two groups of data yields:

$$\frac{d_t}{H_b} = 1.104 \tag{13}$$

Iverson's (1952) laboratory data and field data of Balsillie and Carter (1980) and Balsillie (1980) also suggest that the depth at breaking can be related to the total depth, d_c (d_c is the vertical distance from the top of the wave crest at shore-breaking to the bottom), according to:

$$\frac{d_b}{d_c} = 0.590 \tag{14}$$

as illustrated in Figure 11.

With the same data source used to develop equation (14), d_t and d_c can be related according to:

$$\frac{d_t}{d_c} = 0.529 \tag{15}$$

which is illustrated in Figure 12.

Combination of equations (12) through (15) yields the average result that:

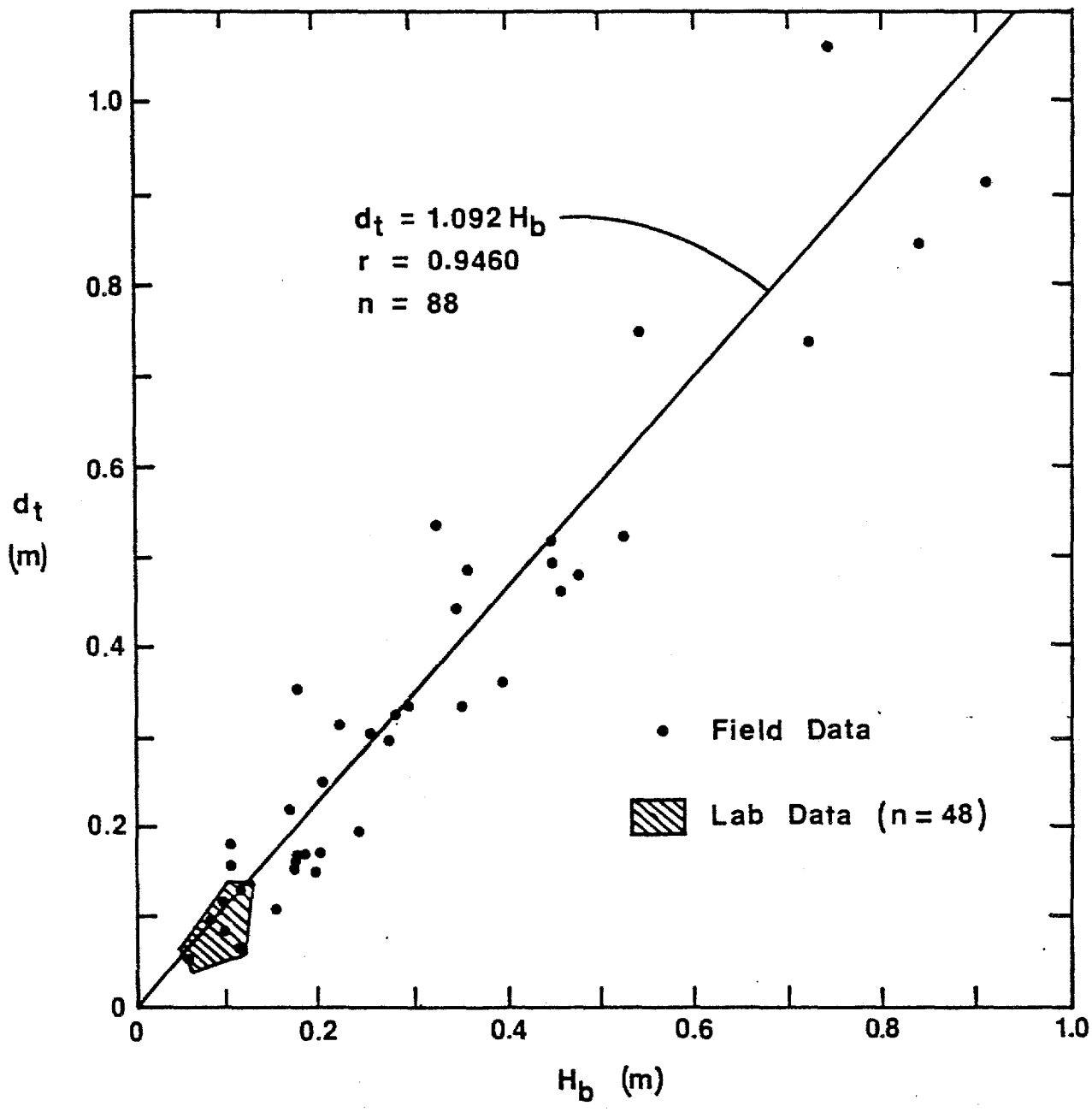

Figure 10. Relationship between the wave height at shore-breaking, H_b, and the trough height just preceding the crest, d_t; field data from Balsillie and Carter (1980) and Balsillie (1980), laboratory data from Iverson (1952).

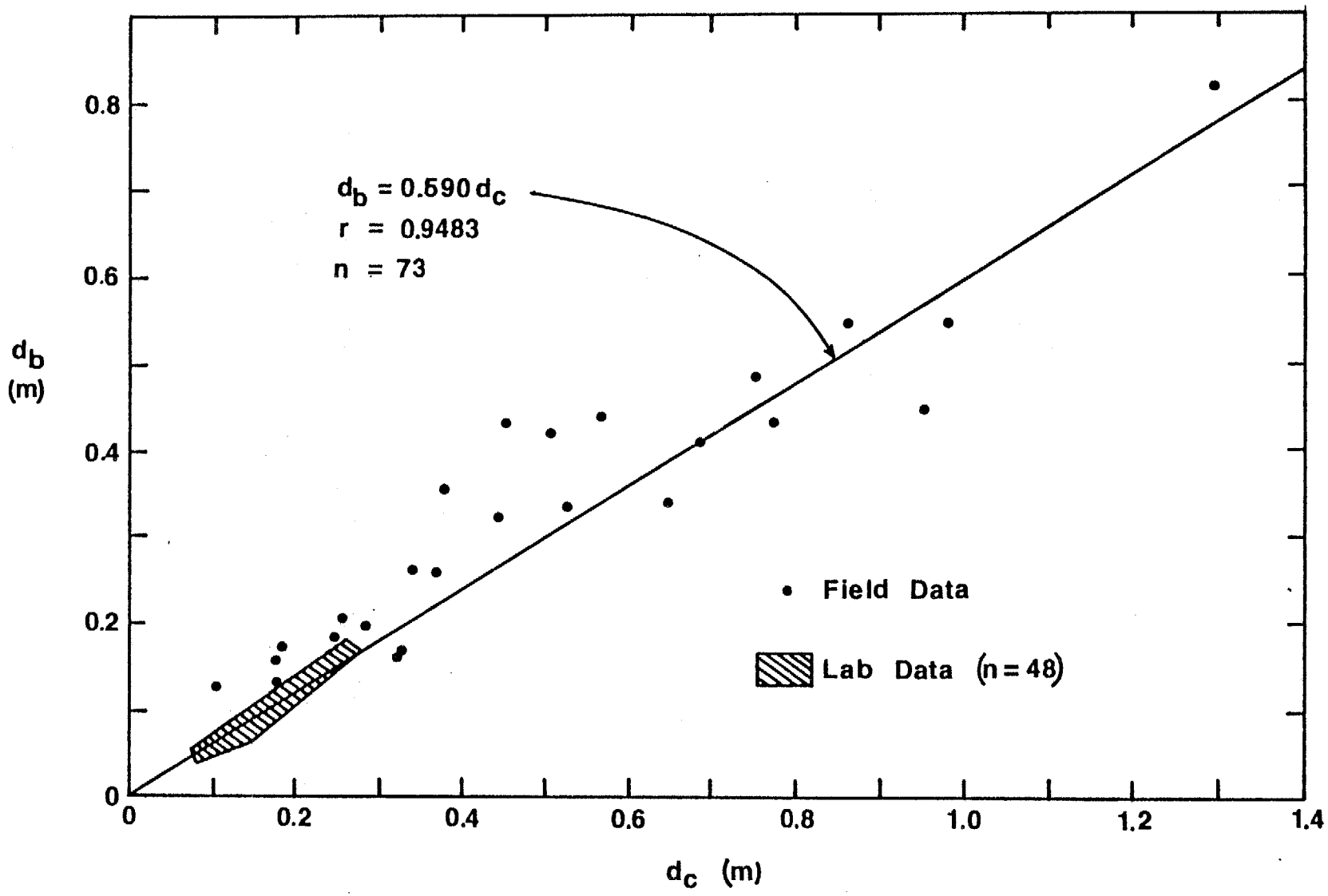

Figure 11. Relationship between the total depth at shore-breaking, d_c, and the water depth at shore-breaking, d_t; field data from Balsillie and Carter (1980) and Balsillie (1980), laboratory data from Iverson (1952).

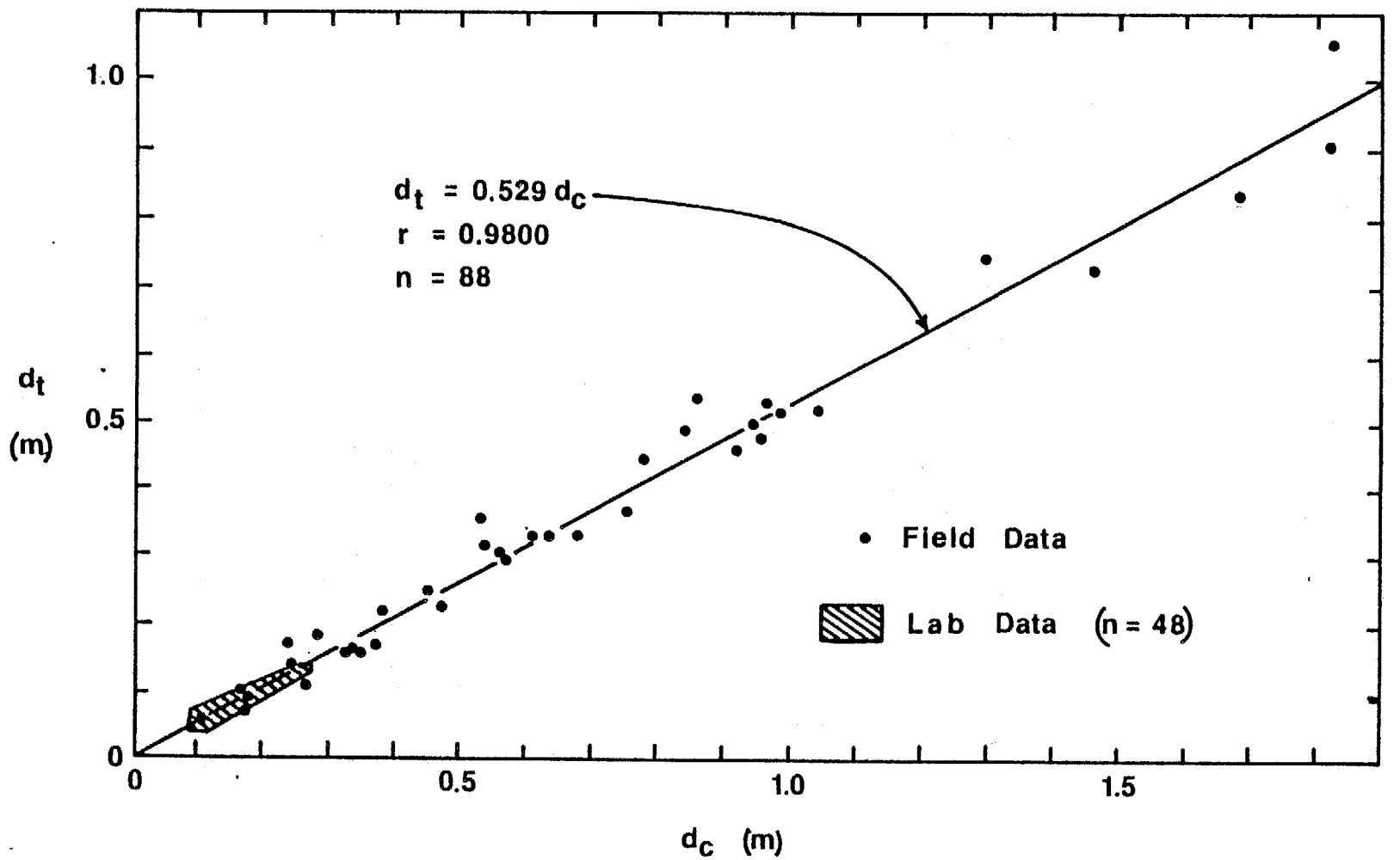

Figure 12. Relationship between the trough height just preceding the wave crest at shore-breaking, d_t, and the total depth at shore-breaking, d_c; field data from Balsillie and Carter (1980) and Balsillie (1980), laboratory data from Iverson (1952).

$$\frac{H_b'}{H_b} = 0.84 \tag{16}$$

which is referenced to the SWL. Hansen (1976) reports that:

$$\frac{H_b'}{H_b} = (0.85)_{MWL} = (0.82)_{SWL} \tag{17}$$

Regardless of the slight discrepancy between the SWL coefficients of equations (16) and (17), Hansen's result has been presented to show that H_b'/H_b is in the mid-eighty percent range and not some significantly large or small value (e.g., the larger breaker index and Bernoulli equation results or those resulting from equation (11), of Figures 2 and 6).

It is appropriate, also, to address the effect of shore-breaker type. The field data of Balsillie and Carter (1980) and Balsillie (1980) represent plunging and spilling type shore-breakers. However, no correlation was found between H_b'/H_b and the shore-breaker type. Weishar (1976) reports the results of a field study using ground photography including the shore-breaker type. Using the established relationship of $d_b/H_b = 1.28$, Weishar's values of H_b'/d_b may be transformed to yield:

$$\left(\frac{H_b'}{H_b}\right)_{Pl} = (0.88)_{MWL} = (0.85)_{SWL} \tag{18}$$

and

$$\left(\frac{H_b'}{H_b}\right)_{Sp} = (0.86)_{MWL} = (0.82)_{SWL} \tag{19}$$

where the subscripts Pl and Sp refer to plunging and spilling shore-breaker types. Weishar (1976) is careful to note, however, that considerable variation

occurred in the data and that the difference between mean values of the last two equations may not be statistically significant. It is concluded, therefore, that while there may be a dependence of the value of H_b'/H_b on the shore-breaker type, sufficient data is not yet available nor work on other methods (e.g., Iwagaki, Sakai, Tsukioka and Sawai, 1974) accomplished to justify such a commitment. Until the matter is resolved, equation (16) shall prevail as least equivocal guidance.

Initial Boundary Condition

The other boundary condition occurs at the beginning of the shore-breaking process. It has been suggested earlier that the maximum value of H'/H in deep water is about 0.64. This maximum value, then, represents forced wave conditions (i.e., the waves are subject to the wind forces from which they were generated, and maximum wave steepness is maintained). A value of less than 0.64 represents free or coasting waves (i.e., the waves are no longer subject to original generating winds, but have left the generation area and have undergone dispersion mechanics). (See Mooers, 1976; Balsillie et al. 1976)

Upon reaching transitional water depths, the bottom slope begins to introduce an additional effect on the value of H'/H. Therefore, H'/H may have a value greater than 0.5 when the wave reaches the point of initiation of the shore-breaking process. When a wave reaching the initiation point is forced, one may expect the progressive increase in the value H'/H to be minimal, provided that bed slope conditions do not change significantly during the shore-breaking process. As seen from the plots of Figure 7, however, for free or coasting waves H'/H does not begin to significantly increase in value until shore-breaking begins, which has been determined to occur when the critical alpha wave peaking depth is encountered. Near the point of initiation of shore-breaking, the value H'/H shall be given the notation (H_i'/H_i) which requires quantification.

From the plots of Figure 7, and those of Balsillie (in manuscript) describing the alpha wave peaking process, it appears that the increase in the wave height above the SWL begins somewhat earlier (i.e., further offshore) in the shore-propagating wave transformation history than does the initiation of the alpha wave peaking process. In terms of design approach, this complication should not be of undue concern, since even though the height of the wave above the SWL might be slightly increasing the total wave height typically is still decreasing ... until the initiation of alpha wave peaking after which H/H_i and H'/H both increase to reach a maximum value at the shore-breaking position. In addition, the initial values of both H/H_i and H'/H (see Figure 7 and Balsillie, in manuscript) are significantly small.

Using the data of Putnam (1945) illustrated in Figure 13, the relative water depth, $(d_i/H_i)''$, indicating where the increase in the wave crest height above the SWL appears to begin may be approximated by:

$$\left(\frac{d_i}{H_i}\right)'' = 1.615 \left(\frac{H_i}{g\ T^2}\right)^{-0.216} \tag{20}$$

The relative depth, $(d_i/H_i)''$, may be related to $(d_i/H_i)'$, the relative depth at which alpha wave peaking is initiated according to:

$$\left(\frac{d_i}{H_i}\right)'' = 3\ e^{0.153 \left(\frac{d_i}{H_i}\right)'} \tag{21}$$

where $(d_i/H_i)'$ is given by Balsillie (in manuscript) by:

$$\left(\frac{d_i}{H_i}\right)' = \frac{d_b}{H_b} - \frac{\pi}{2} \ln \left[\tanh \left(65\ \frac{H_i}{g\ T^2}\right)\right] \tag{22}$$

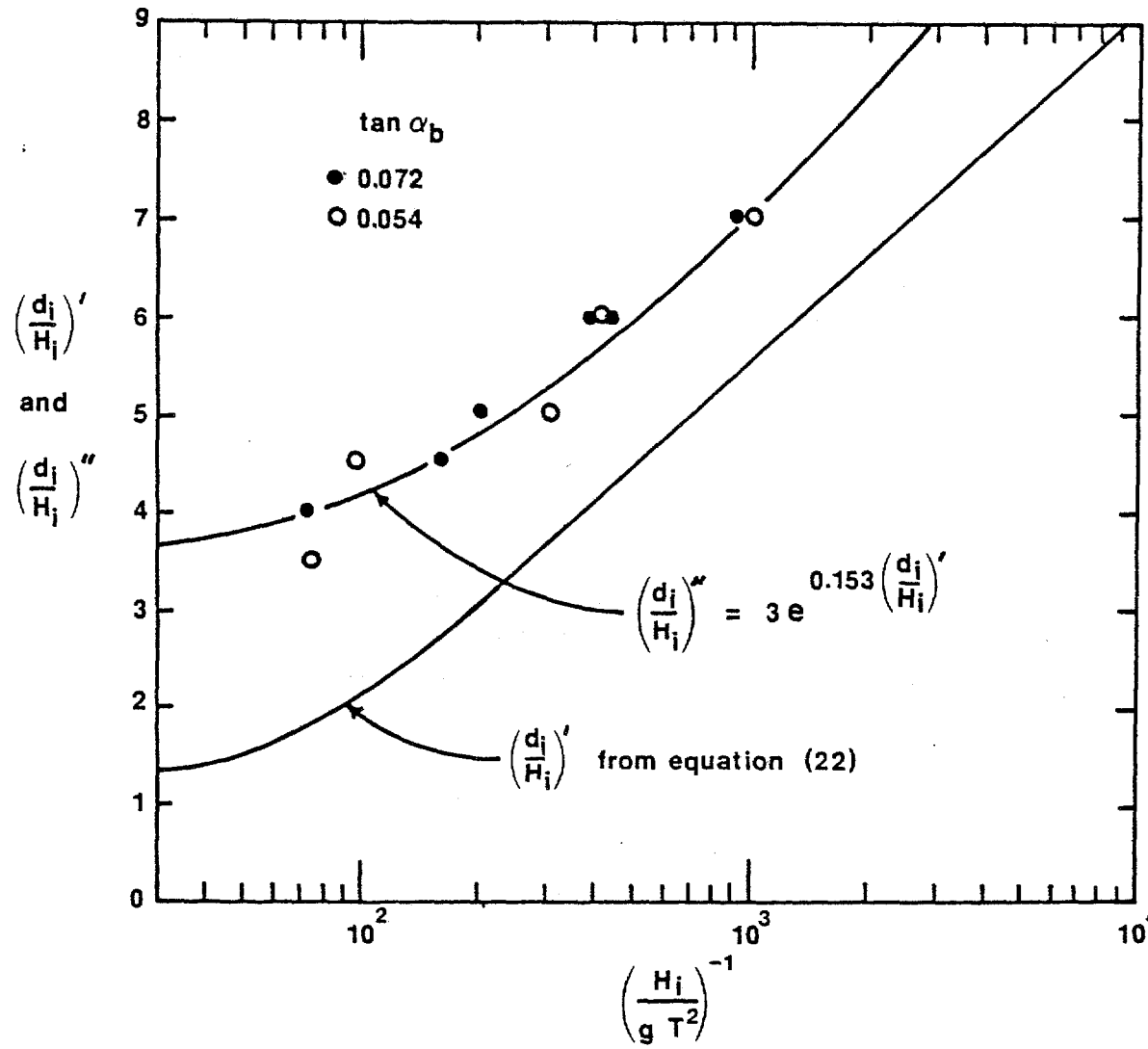

Figure 13. Relationship for determination of the incident relative depth $(d_i/H_i)''$ at which (H_i'/H_i) occurs in terms of the relative depth $(d_i/H_i)'$ where alpha wave peaking is initiated.

The wave steepness parameter at the two points can also be expected to be somewhat different in value. Again, the data of Putnam (1945) indicates a slight difference (Figure 14) that may be approximated by:

$$\left(\frac{H_i}{g\ T^2}\right)'' = 1.462\left(\frac{H_i}{g\ T^2}\right)^{1.055} \tag{23}$$

where $(H_i/(g\ T^2))$ is the wave steepness parameter at initiation of alpha wave peaking, and $(H_i/(g\ T^2))''$ occurs at $(d_i/H_i)''$.

The relative incident wave height, (H_i'/H_i) , conforming to the relative depth conditions of equations (20), (21) and (23), illustrated in Figure 15, may be given by:

$$\frac{H_i'}{H_i} = 0.5 + \left[1.25\left(\frac{H_i^2}{g\ d\ T^2}\right)''\right]^{0.5} \tag{24}$$

in which $(H_i^2/(g\ d\ T^2))''$ occurs at $(d_i/H_i)''$.

In addition, it becomes of value to be able to predict the magnitude of H'/H at the point of initiation of alpha wave peaking (i.e., at $(d_i/H_i)'$). based on numerical analyses from the results of this work and that of Balsillie (in manuscript), the value of H'/H at the initiation of alpha wave peaking, given the notation $(H_i'/H_i)_\alpha$, is suggested to be given by:

$$\left(\frac{H_i'}{H_i}\right)_\alpha = 0.54 + 10.34\left(\frac{H_i}{g\ T^2}\right)^{1.014} \tag{25}$$

or by

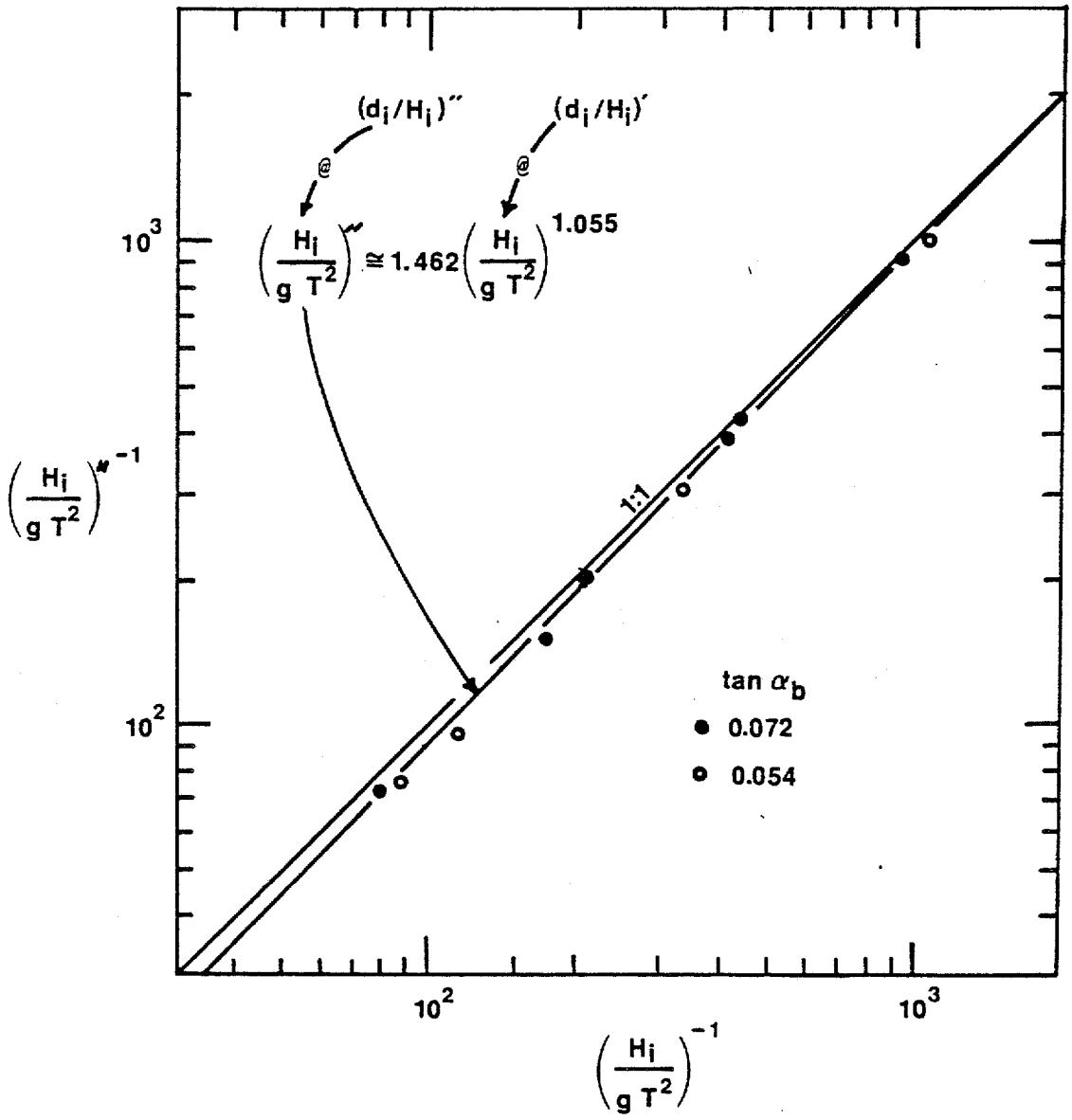

Figure 14. Wave steepness parameter at $(d_i/H_i)''$ related to the wave steepness parameter at $(d_i/H_i)'$.

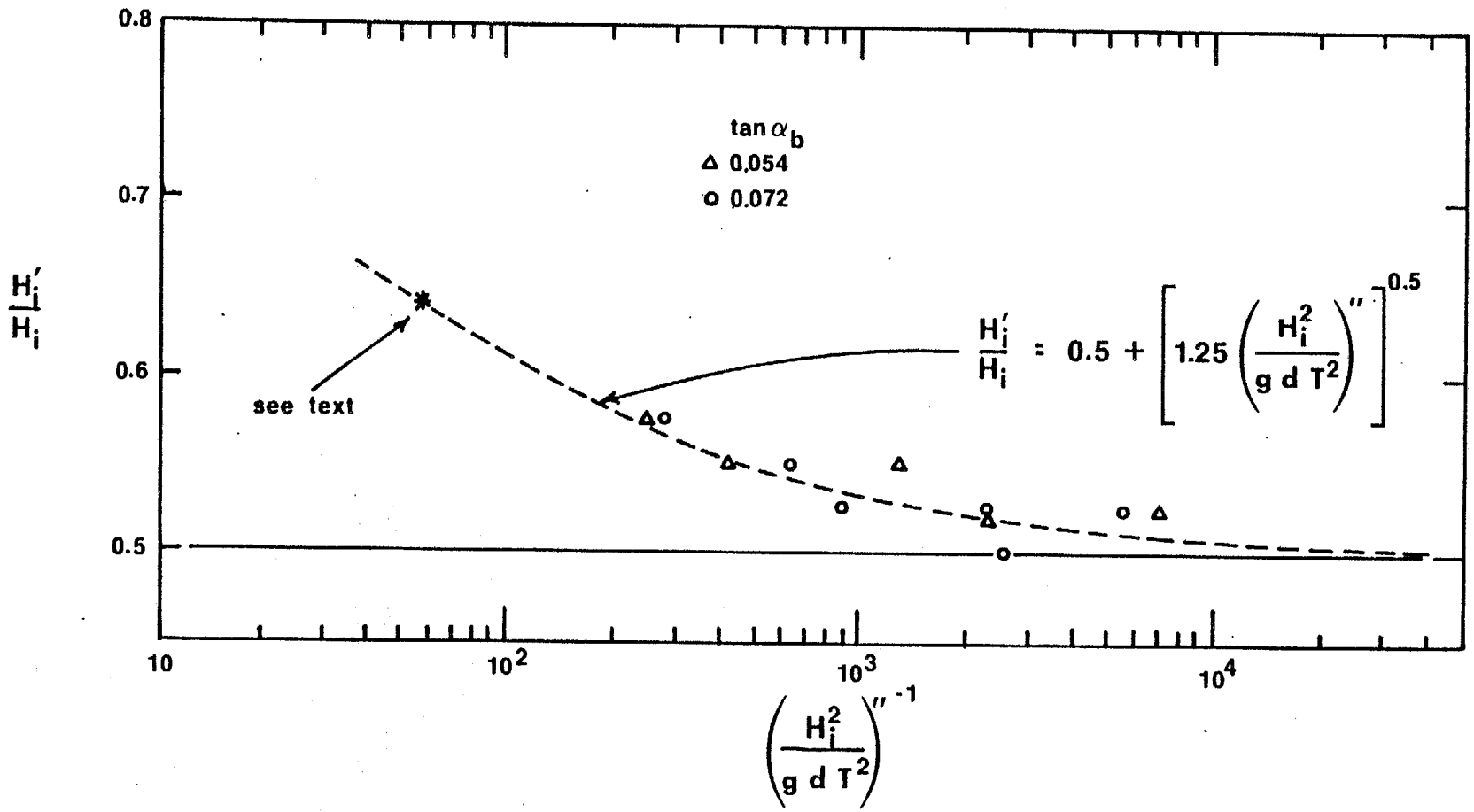

Figure 15. Relationship for prediction of the incident value of H'/H.

$$\left(\frac{H_i'}{H_i}\right)_\alpha = 0.84 - 0.307 \left\{\tanh\left[0.3\left(\frac{d_i}{H_i} - \frac{d_b}{H_b}\right)\right]\right\}^{0.309} \tag{26}$$

It is interesting to note that where limiting conditions are imposed, equation (24) provides consistent results. Suppose that a step profile occurs where a wave, initially in deep water, suddenly encounters an abrupt slope change and it must shore-break. Suppose, also, that the wave in deep water is fully forced so that $(H_o'/H_o) = (H_i'/H_i) = 0.64$. For such conditions, from equation (24), $(H_i^2/(g\ d\ T^2))^{-1} = 56.4$. From another viewpoint, where $H_b/d_b = 1/1.28 = 0.78$ and $H_i/(g\ T^2) = 1/(14\ \pi)$, then $(H_i^2/(g\ d\ T^2))^{-1} = 63.8$ and $(H_i'/H_i) = 0.648$. Both solutions are close, and represented by the asterisk in Figure 15.

Also, where the maximum possible value of (H_i'/H_i) in equation (25) is 0.84, representing forced wave conditions in shallow water, the maximum value of $(H_i/(g\ T^2)) = 0.305$. Where $H_b/L_b = 0.79\ (H_b/(g\ T^2))^{0.5}$ given by Balsillie (in manuscript), then by substitution $(H/L)_{max}$ for shallow water waves becomes 0.138 or 1/ 7.2. This result is in accordance with the Michell (1893) criterion of 1/7.

Prediction of H'/H Transformation

Incorporation of the preceding boundary conditions leads to the following development describing the H'/H transformation during the shore-breaking process. The general equation is given by:

$$\frac{H'}{H} = \frac{H_b'}{H_b} - \Phi_3 \left\{\tanh\left[\Phi_1\left(\frac{d}{H} - \frac{d_b}{H_b}\right)\right]\right\}^{\Phi_2} \tag{27}$$

where $d_b/H_b = 1.28$, and:

$$\Phi_1 = \frac{e}{(d_i/H_i)'' - 1.28} \simeq \frac{2.7183}{(d_i/H_i)'' - 1.28} \tag{28}$$

in which e is the Naperian constant,

$$\Phi_2 = -0.384 - 0.2 \ln\left(\frac{H_i}{g\,T^2}\right)'' \tag{29}$$

and

$$\Phi_3 = \frac{H_b'}{H_b} - \frac{H_i'}{H_i} \tag{30}$$

where from equation (16) $H_b'/H_b = 0.84$, (H_i'/H_i) is given by equation (24) and $(d_i/H_i)''$ is given by equation (21).

Equation (27) is plotted in Figure 7 as the dash-dot-dash curves. It is to be noted from these plots that equation (27) appears to more successfully represent initial values of H'/H (i.e., incident waves with larger wave steepness values tend to have larger initial H'/H values) than does equation (7). There is, however, discrepancy between measured and predicted values of H'/H as the shore-breaking position is closely approached. Putnam's (1945) laboratory data tend to consistently underestimate the value of H'/H very near and at the shore-breaking position. One must recall, however, that the behavior of the curve predicted by equation (27) is determined by the terminal boundary condition that $H_b'/H_b = 0.84$, which is based on prototype wave data and results from other investigations. Hence, laboratory conditions or measurement techniques may account for the apparently low values of Putnam's data near shore-breaking. While the terminal boundary condition must surely be refined by future research efforts, equation (27) would appear to provide a satisfactory and, certainly, a useful method for predicting H'/H during the shore-breaking process.

Equation (27) is also tested using the prototype laboratory waves (Figure 16) and field hurricane waves (Figure 17) reported by Bretschneider

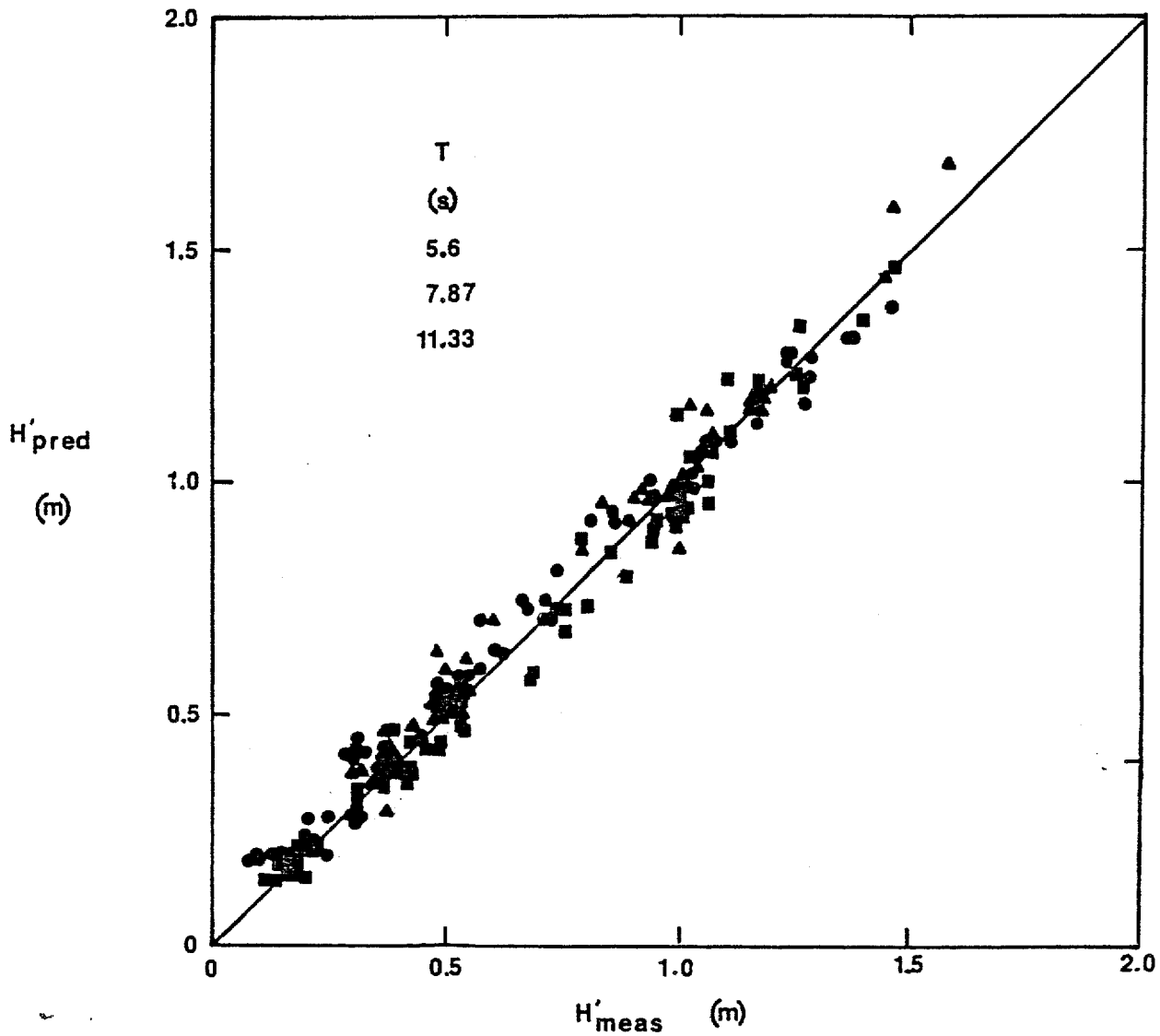

Figure 16. Predicted H' from equation (27) versus prototype Beach Erosion Board wave tank data reported by Bretschneider (1960).

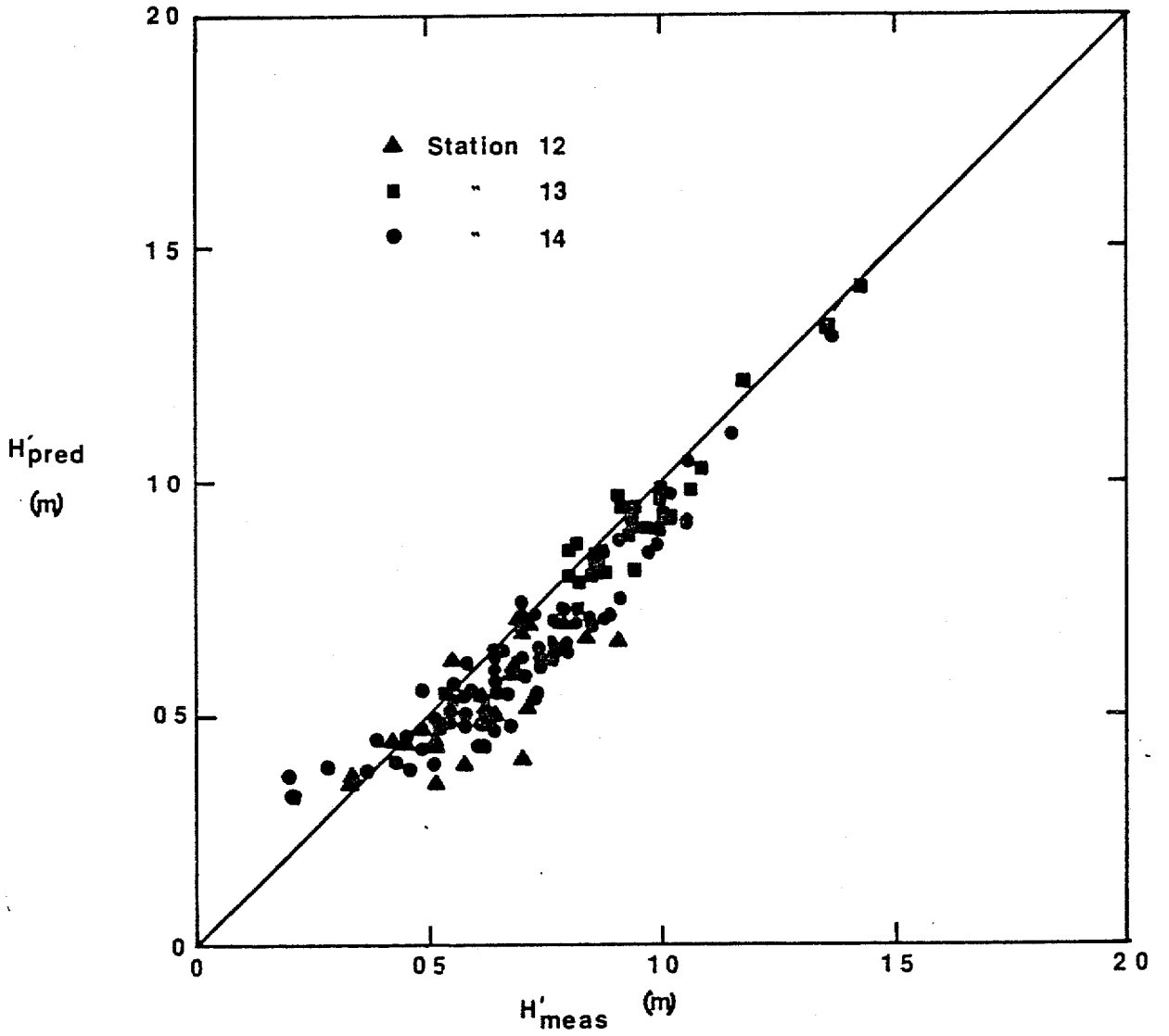

Figure 17. Predicted H' from equation (27) versus Lake Okeechobee hurricane wave data reported by Bretschneider (1960).

(1960). Figure 16 indicates very good agreement. Figure 17, however, indicates that equation (17) underestimates H'/H. One must keep in mind that the measured data represent the maximum wave height occuring during one-minute recording periods, not the average wave height. Therefore, one would expect equation (27) to underestimate the measured data, and that the plotted line represents an expected upper limit as also found by Bretschneider (1960).

Based on the success of equation (27), the value of H'/H as a function of $H/(g\ T^2)$ and $d/(g\ T^2)$ is given by Figure 18 for transitional and shallow water depths. Figure 18, then, provides an alternative to the nomographic approach originally proposed by Bretschneider (1960).

CLOSURE

Two basic wave processes during shore-breaking have been identified as: 1. the height of the wave crest tends to increase to reach a maximum at the shore-breaking position, and 2. the amount of the wave crest lying above the still water level increases, again, reaching a maximum at shore-breaking. The former process is described by a companion paper (Balsillie, in manuscript), the latter has been investigated in the present work.

Available field and laboratory data indicate that at shore-breaking, the relative amount of the breaker crest lying above the still water level, H_b'/H_b, is 0.84 where H_b is the shore-breaker height and H_b' is the amount lying above the still water level. In addition, during the shore-breaking process the amount of H' tends to rapidly and progressively increase. This increase, which is important to consider in such concerns as the determination of uplift pressures, is quantified by equation (27).

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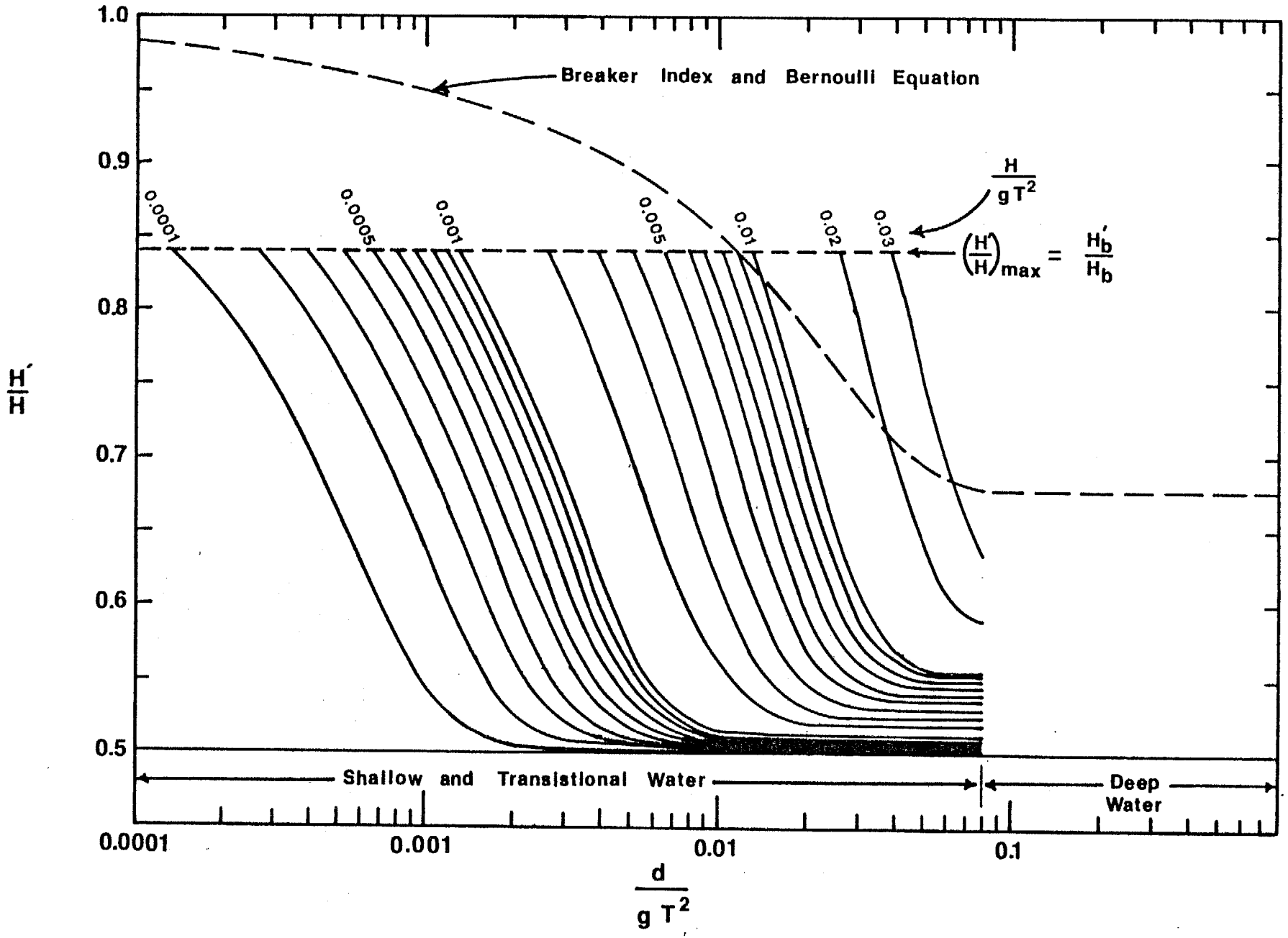

Figure 18. Ratio of wave crest elevation above the still water level to wave height from equation (27).

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