A Simple Method for Measuring Straightness of Coordinate Measuring Machines

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U.S. DEPARTMENT OF COMMERCE
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Introduction

Straightness errors contribute significantly to the total error budget of coordinate measuring machines. There are two straightness error parameters for each axis; horizontal and vertical straightness. According to Bryan [1], straightness error is "a non-linear movement of the machine axis that an indicator sees when it is either stationary reading against a perfect straight edge supported on a moving slide or moved by the slide along a perfect straight edge which is stationary." The MITF definition for straightness error is that "Carriage straightness error is the motion of a carriage, designed for linear translation, perpendicular to the intended motion axis. Two types of straightness error are defined: Type M straightness, which is the movement one would measure with a stationary indicator against a perfect straightedge supported on the moving carriage and aligned with its motion axis; and Type F straightness, which is that movement measured by an indicator attached to the carriage reading against a fixed straightedge similarly aligned" [2]. Thus, straightness error can be determined as the deviation of measurement data from a straight line.

For the purpose of calibrating coordinate measuring machines, a laser interferometer system equipped with straightness optics is often used to measure straightness as is a mechanical straightedge and indicator. In addition to the relative high cost of a laser interferometer system, it needs and experienced operator to perform the measurements.
A simple and rapid method for measuring the straightness of coordinate measuring machines was developed using the ball bar.

Theoretical Approach

As shown in Figure 1, if point D moves at a constant distance from another fixed point 0 forming a circular path in the X-Y plane for example, then the difference between the exact Y-location of this point from the Y-location as measured by the machine represent the Y component of motion error at that point.

In using the ball bar, the fixed end was clamped at point 0 in the X-Y plane of the coordinate measuring machine and the free ball moved along the arc $A_1 B_1$.

If point D is a general position of the free ball, the true distance ($Y_t$) between point D and line AB can be obtained if the true length, L of the ball bar is known. Thus, by obtaining the actual distance (Y) between point D and line AB, the deviation of Y from $Y_t$ represent the motion error at point D. Line AB is the reference standard line. From the mathematical model of the machine, the Y motion can be expressed by

$$Y = Y_t - \delta_y(Y) - \delta_y(X) - X \cdot \varepsilon_z(Y) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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Thus,

$$\delta_y(x) = y_t - y - \delta_y(Y) - x \cdot \epsilon_z(Y) \quad \ldots \quad (2)$$

From Figure 1,

$$y_t = L - \sqrt{L^2 - (R - X_D)^2} \quad \ldots \quad (3)$$

There was one assumption for this analysis, that is the machine scale errors have negligible values. If this assumption were not valid, then a step gage would be used to measure the scale errors of the machine following the procedure outlined in the B89.1.12 standard [3].

Within a small range, the rotation of the Y-axis slide about the Z-axis can be expressed as

$$\epsilon_z(Y) = K \cdot Y \quad \ldots \quad (4)$$

where $K$ is a constant.

Substituting equations (3) and (4) into equation (1), then

$$y = L - \sqrt{L^2 - (R - X_D)^2} - \delta_y(Y) - K \cdot X \cdot Y \quad \ldots \quad (5)$$

By aligning the data such that straightness error is zero at both ends, then at point $B_1 (X_e, Y_e)$; $\delta_y(X_e) = 0$

Using equations (2), (3), and (4) and substituting $X_D = X_e$, $Y = Y_e$, and $x = X_e$, then,

$$L - \sqrt{L^2 - (R - X_e)^2} - Y_e - \delta_y(Y_e) - K \cdot X_e \cdot Y_e = 0 \quad \ldots \quad (6)$$

Therefore,
Substituting equations (4), (5), and (7) into equation (2), we get the general expression for \( \delta_y(x) \) at any point,

\[
\delta_y(x) = Y_t - Y - \delta_y(Y) - K \cdot X \cdot Y \quad \ldots \ldots \ldots \ldots (8)
\]

If the assumption of zero scale errors is valid, then \( \delta_y(Y) = 0 \), and for any point \((X,Y)\), the \( Y \)-straightness of \( X \) is given by:

\[
\delta_y(x) = L - \sqrt{L^2 - (R - X)^2} - Y \cdot \left( \frac{L \cdot \sqrt{L^2 - (R - X)^2} - Y_e}{X_e \cdot Y_e} \right) \cdot X \cdot Y \quad \ldots \ldots (9)
\]

Experiments & Results

A ball bar length of about 502.00 mm was used for the experiments. The fixed end of the ball bar was clamped at a height of 380 mm above the machine table and the \( x \) and \( y \) coordinates for the fixed ball were 400 and 30 mm respectively. The measuring range for the experiments, that is the distance between points \( A \) and \( B \) in Figure 1, was 600 mm. Straightness errors were obtained from equation (9).

Coordinates of points \( A_1 \) and \( B_1 \) \((X_e, Y_e)\) were determined from the CMM display. The \( x \)-coordinate from point \( A_1 \) was set to zero at the beginning of the measurements and point \( B_1 \) was located such that \( A_1 \) and \( B_1 \) have the same \( y \)-coordinate \((Y_e)\). Distance \( R \) was obtained by dividing \( X_e \) by 2. A computer program using HP Basic was written to compute values for \( \delta_y(x) \) at intervals of 30.00 mm starting from point \( A_1 \) and ending at point \( B_1 \). The ball bar length \( L \) was accurately
measured. The ball bar length $L$, $X_e$, and $Y_e$ were input to the computer program, and $x$ and $y$ coordinates were entered to the program at each measurement point. The program then performed calculations for $\delta_y(X)$ and printed out straightness values after aligning the data such that $\delta_y(X) = 0$ at both ends. This procedure was repeated ten times.

Figures 2 through 11 show the obtained results for the ten runs, all at the same position and under the same conditions. Measuring the ball bar length, $L$ is very critical and should be measured with special care.

**Accuracy of the Method**

In order to check the accuracy or applicability of using the ball bar for the measurement of straightness errors of coordinate measuring machines a straight edge was used to measure straightness of the same machine at the same positions where ball bar measurements were made. This process also was run for 10 times.

Results of straight edge measurements are shown in Figures 12 through 21 which are similar to those obtained using the ball bar. This proves the applicability of the ball bar for cmm straightness measurement.

The average straightness curves for both the ball bar and straightedge methods are shown in Figures 22 and 23 respectively. Tables 1 and 2 show the average straightness values (of 10 runs), the standard deviation of each point, and the process standard deviation for the ball bar and straight edge respectively.
Further Statistical Analysis

When two population means are to be compared, it is usually their difference that is important, rather than their absolute values. A statistically acceptable estimate of the difference in population means is the difference in sample means.

The distribution of the difference in average straightness values between the ball bar results and straight edge results should follow a distribution with a mean $\bar{x}$ and a standard deviation $\sigma$.

\[ \bar{x} = \delta_{\text{av}(x)_{bb}} - \delta_{\text{av}(x)_{se}} \]  

where

$\delta_{\text{av}(x)_{bb}}$ is the average of the average straightness values in table 1 for the ball bar.

$\delta_{\text{av}(x)_{se}}$ is the average of the average straightness values in table 2 for the straightedge.

\[ \sigma^2 = \frac{1}{(n_1+n_2-2)} \left( \sigma_{bb}^2 + \sigma_{se}^2 \right) \]  

\( n_1 = n_2 = \text{number of data points} \)

$\sigma_{bb} = \text{standard deviation for the ball bar method. Shown in table 1.}$

$\sigma_{se} = \text{Standard deviation for the straightedge method. Shown in table 2.}$

The composite curve for the differences in the average straightness values between the two methods is shown in Figure 24. The average straightness values and standard deviation for that curve are $-0.7 \mu m$ and $0.9 \mu m$ respectively.

A highly useful means of examining the applicability of the proposed method for measuring straightness of coordinate measuring machines is residual analysis. A residual is the difference between
the measured straightness value and the average value. Average
straightness values are in agreement with those obtained using a
straightedge. A residual, $e_i$, is given by:

$$e_i = \delta_y(x_i) - \delta_{Yav}(x_i)$$

(12)

where $\delta_{Yav}(x_i)$ is the average of ten measured
straightness values at point $x_i$.

The histograms of residuals are shown in Figure 25. Residuals
should have a distribution with a mean of about zero and the
distribution should be reasonably normal. The two distributions for
the straightedge method and ball bar method satisfy this requirement
except, as shown in Figure 25, the histogram of residuals for the
ball bar method has a mean of 1.0 $\mu$m. This shift is caused by the
assumption that the machine has zero scale errors. The mean of the
histogram will be shifted back to zero if the scale errors are in the
range of -1.0 or -2.0 $\mu$m. This is true for the machine used for
taking the measurements.

Summary

The ball is a simple, easy, and rapid method that can be applied
to the calibration of coordinate measuring machines. Using the ball
bar for cmm straightness measurement was proved to be an applicable
and simple method.
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<th>Std. Deviation (mm)</th>
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<td>0.0000</td>
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Standard Deviation = 0.0028 mm
### TABLE 2

**AVERAGE STRAIGHTNESS VALUES FOR STRAIGHT-EDGE METHOD**

<table>
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<tr>
<th>Pt. #</th>
<th>Average (mm)</th>
<th>Std. Deviation (mm)</th>
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<tbody>
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<tr>
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*Standard Deviation = .0042*
References


Figure 1 Principle of Using the Ball Bar for CMM Straightness Measurement
Figure 2 Y - Straightness of X - Axis Using The Ball Bar
Figure 3: Y - Straightness of X - Axis Using The Ball Bar

ERROR (MM)
0.008 0.006 0.004 0.002 0.000 0.08 0.04 0.02
Figure 4  Y - Straightness of X - Axis
Using The Ball Bar
Figure 5

Y - Straightness of X-Axis Using The Ball Bar
Figure 7: Y - Straightness of X-Axis Using The Ball Bar
Figure 9  Y - Straightness of X - Axis
Using The Ball Bar
Figure 10  Y - Straightness of X - Axis Using The Ball Bar
Figure 12  Y - Straightness of X-Axis Using Straight Edge
Figure 13  Y - Straightness of X - Axis Using Straight Edge
Figure 14  Y - Straightness of X - Axis
Using Straight Edge
Figure 15  Y - Straightness of X - Axis Using Straight Edge
Figure 19  Y - Straightness of X - Axis Using Straight Edge
Figure 20  Y - Straightness of X - Axis Using Straight Edge
Figure 22 Y - Straightness of X - Axis Using The Ball Bar (Average)
Figure 23  Y - Straightness of X - Axis  
Using Straight Edge (Average)
Figure 24  Y - Straightness of X - Axis Differences Between Averages
Figure 25

HISTOGRAM OF RESIDUALS

FREQUENCY

STRAIGHTNESS VALUES
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Ahmad K. Elshennawy, Fang-Sheng Jing and Robert J. Hocken

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Ball bar, Coordinate Measuring Machine, Laser Interferometer, Machine Axis Straightness errors, Vertical Straightness